PRINCIPLE OF INCLUSIVE AND EXCLUSIVE 8m. (18)

If A and B are not mutually exclusive or If A and B are finite sets then.

→ $|A\cup B| = |A| + |B| - |A\cap B|$

→ $|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |B\cap C| - |A\cap C| + |A\cap B\cap C|$

→ $|A\cup B\cup C\cup D| = |A| + |B| + |C| + |D| - |A\cap B| - |B\cap C| - |C\cap D| - |D\cap A| - |A\cap C| - |B\cap D| + |A\cap B\cap C| + |A\cap B\cap D| + |A\cap C\cap D| + |B\cap C\cap D| - |A\cap B\cap C\cap D|$

REMARK. ① Either or $\Rightarrow |A\cup B|$; Neither nor, none $\Rightarrow |\overline{A\cup B}|$

② $|U| = |A| + |\bar{A}|$ (or) $|U| = |A\cup B| + |\overline{A\cup B}|$

VERIFY THE PRINCIPLE OF INCLUSION & EXCLUSION FOR THE DAY.

① Let $A = \{a,b,c,d,e\}$, $B = \{c,e,f,h,k,m\}$.

Sol: Verify: $|A\cup B| = |A| + |B| - |A\cap B|$.

$|A| = 5$; $|B| = 6$, $A\cap B = \{c,e\}$, $|A\cap B| = 2$, $|A\cup B| = 9$.

L.H.S $= |A\cup B| = 9$; R.H.S $= 5+6-2 = 9$. ∴ L.H.S = R.H.S.

② $A = \{a,b,c,d,e\}$, $B = \{a,b,e,g,h\}$, $C = \{d,b,e,g,h,k,m,n\}$

Sol: Verify: $|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$.

$|A| = 5$, $|B| = 5$, $|C| = 8$, $A\cap B = \{a,b,e\} \Rightarrow |A\cap B| = 3$.

$A\cap C = \{b,d,e\} \Rightarrow |A\cap C| = 3$; $B\cap C = \{b,e,g,h\} \Rightarrow |B\cap C| = 4$.

$(A\cap B\cap C) = \{b,e\} \Rightarrow |A\cap B\cap C| = 2$; $(A\cup B\cup C) = \{a,b,c,d,e,g,h,k,m\}$

$|A\cup B\cup C| =$ L.H.S $= 10$. ; R.H.S $= 5+5+8-3-3-4+2 = 20-10 = 10$

∴ L.H.S = R.H.S.

③ $A = \{x \mid x$ is an +ve integer $< 8\}$

$B = \{x \mid x$ is an +ve int $2 \le x \le 5\}$.

Sol: $A = \{1,2,3,4,5,6,7\}$, $B = \{2,3,4,5\}$

$A\cap B = \{2,3,4,5\}$, $A\cup B = \{1,2,3,4,5,6,7\}$

VERIFY: $|A\cup B| = |A| + |B| - |A\cap B|$

L.H.S $= |A\cup B| = 7$; R.H.S $= 7+4-4 = 7$

∴ L.H.S = R.H.S

④ $A = \{x \mid x$ is a +ve integer & $x^2 \le 16\}$

$B = \{x \mid x$ is a -ve integer & $x^2 \le 25\}$.

Sol: $A = \{1,2,3,4\}$ & $B = \{-1,-2,-3,-4,-5\}$.

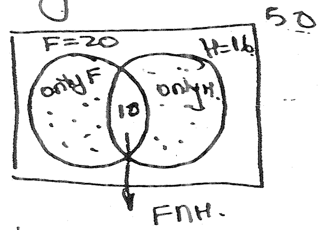
$A\cap B = \phi$, $A\cup B = \{-5,-4,-3,-2,-1,1,2,3,4\}$.

Verify: $|A\cup B| = |A| + |B| - |A\cap B|$.

L.H.S $= |A\cup B| = 9$; R.H.S $= 4+5-\phi = 9$ ∴ L.H.S = R.H.S

⑤ In a class of 50 students, 20 students play football, & 16 students play Hockey. It is found that 10 students play both the games. Use the algebra of sets to find out the number of students who play neither.

Sol: Let $U = \{\text{Students in the class}\} \Rightarrow |U| = 50$.



$A = \{\text{Students play football}\} \Rightarrow |A| = 20$.

$B = \{\text{Students play Hockey}\} \Rightarrow |B| = 16$.

$A \cap B = \{\text{students play both games}\} \Rightarrow |A \cap B| = 10$.

TO FIND: No: of Students neither Play hockey or football $\} = |\overline{A \cup B}| = |U| - |A \cup B| \rightarrow ①$

W.K.T $|A \cup B| = |A| + |B| - |A \cap B| = 20 + 16 - 10 = 26$.

$\therefore$ ① $\Rightarrow |\overline{A \cup B}| = 50 - 26 = 24$.

$\therefore$ 24 students play neither of games.

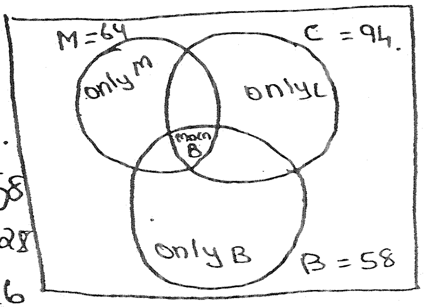
---

⑥ In a survey of 260 college students, the following data are obtained: 64 had taken a mathematics course, 94 had taken a Computer science course, 58 had taken business course, 28 had taken both a mathematics & a business course, 26 had taken a mathematics & Computer course, 22 had taken both Computer science & a business course, 14 had taken all three courses.

(i) How many students were surveyed who had taken none of the three courses?

(ii) Of the students surveyed, how many had taken only a Computer science courses.

Sol: Let $U = \{\text{students in a College}\} \Rightarrow |U| = 260$



$M = \{\text{Students taken Maths Course}\} \Rightarrow |M| = 64$.

$C = \{\text{Students taken Computer Course}\} \Rightarrow |C| = 94$.

$B = \{\text{Students taken Bussiness Course}\} \Rightarrow |B| = 58$

$M \cap B = \{\text{Students taken Maths \& Bussiness}\} \Rightarrow |A \cap C| = 28$

$M \cap C = \{\text{Students taken Maths \& Computer}\} \Rightarrow |M \cap C| = 26$

$C \cap B = \{\text{Students taken Computer and Bussiness}\} \Rightarrow |C \cap B| = 22$.

$M \cap B \cap C = \{\text{Students taken all the three Courses}\} \Rightarrow |A \cap B \cap C| = 14$.

(i) To Find: No: of students taken None of the 3 Courses $\}$ $|\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C| = ? \rightarrow ①$

W.K.T $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
$= 64 + 94 + 58 - 26 - 28 - 22 + 14 = 154$.

① $\Rightarrow |\overline{A \cup B \cup C}| = 260 - 154 = 106$. $\therefore$ 106 students had taken none of the courses.

(ii) TO FIND: No: students taken only Computer Scien Course = ?

No: of students taken only a Computer Science $= 94 - 12 - 14 - 8 = 60$

(7) Cricket, Hockey & football are played in a city. A survey conducted yields, 40% play cricket, 32% Hockey, 28% football, 16% cricket and Hockey, 10% cricket & football, 8% Hockey & football, 4% all the three. Determine (19)

(i) what % play only football.

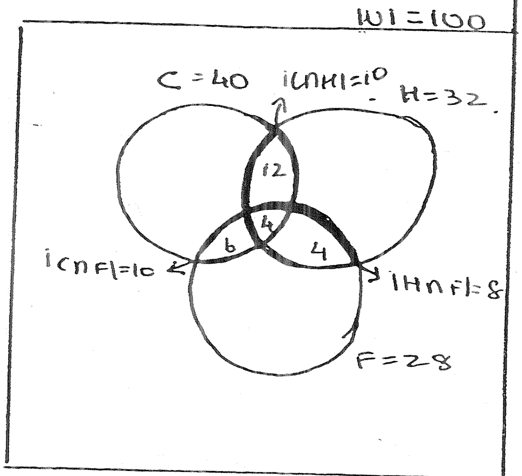
(ii) what % play atleast one of the 3 games.

(iii) what % play either cricket or football

Sol: $|U| = 100$, $|C| = 40$, $|H| = 32$, $|F| = 28$

$|C \cap H| = 16$, $|C \cap F| = 10$, $|H \cap F| = 8$

$|H \cap F \cap C| = 4$.



(i) To find: % PLAY ATLEAST ONE OF THE 3 GAMES

$|H \cup F \cup C| = |H| + |F| + |C| - |H \cap F| - |F \cap C| - |C \cap H| + |H \cap F \cap C|$.

$= 32 + 28 + 40 - 8 - 10 - 16 + 4 = 70$.

$\therefore$ 70% Play atleast one of 3 games

(ii) TO FIND: % PLAY ONLY FOOTBALL

Only football $= 28 - 6 - 4 - 4 = 14\%$.

$\therefore$ 14% play only football.

(iii) TO FIND: % PLAY EITHER CRICKET OR FOOTBALL.

$|C \cup F| = |C| + |F| - |C \cap F| = 40 + 28 - 10 = 58$

$\therefore$ 58% play either cricket or football

(8) A survey of 100 students w.r.t their choice of the ice cream flavours vanilla, chocolate and strawberry shows that 50 students like vanilla, 43 like chocolate, 28 like strawberry, 13 like vanilla and chocolate, 11 like chocolate & strawberry, 12 like strawberry and vanilla & 5 like all of them. Find the number of students who like.

(i) Vanila only (ii) chocolate only (iii) strawberry only (iv) chocolate but not strawberry (v) chocolate and strawberry but not vanilla (vi) Vanila or chocolate but not strawberry. Also find the number of students who do not like any of these flavours.

Given.

$|S \cap V \cap C| = 5$, $|U| = 100$, $|V \cap C| = 13$, $|C \cap S| = 11$, $|S \cap V| = 12$, $|V| = 50$

$|C| = 43$, $|S| = 28$.

(a) No: of students who like Vanilla only $= 30$

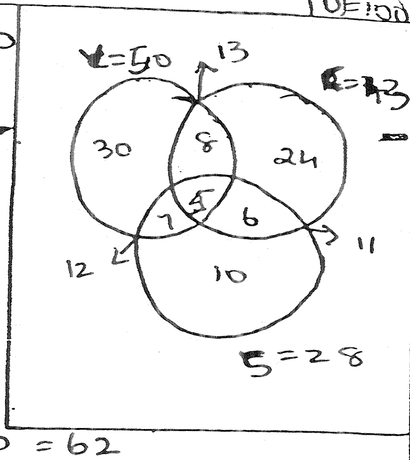
(b) No: of students who like chocolate only $= 24$.

(c) No: of students who like strawberry only = 10

(d) No: of students who like chocolate but not strawberry } = 24 + 8 = 32

(e) No: of students who like chocolate & strawberry but not vanilla } = 6.

(f) No: of students who like vanilla or chocolate but not strawberry = 24 + 8 + 30 = 62



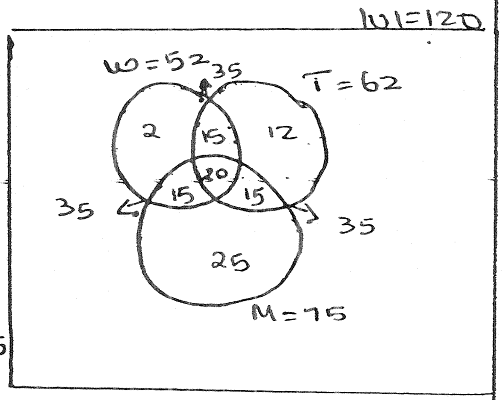
(g) No: of students who donot like any of these flavour } = $|\overline{A \cup B \cup C}| = ?$

$|\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C| = |U| - [|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|]$

$= 100 - [50 + 43 + 28 - 13 - 11 - 12 + 5] = 100 - 90 = 10.$

(9) In a survey of 120 passengers an airline found that 52 enjoyed wine with their meals, 75 enjoyed mixed drinks & 62 enjoyed iced tea. 35 enjoyed any given pair of these beverges and 20 passengers enjoyed all of them. Find the no: of passengers who enjoyed (i) Only tea (ii) Only one of the three (iii) Exactly 2 of the breverges (iv) None of the drinks.

Sol: $|T \cap W \cap M| = 20$, $|W| = 52$, $|M| = 75$, $|T| = 62$, $|W \cap T| = 35$, $|W \cap M| = 35$, $|M \cap T| = 35$, $|U| = 120$.



(i) No: of passengers who enjoyed only Tea } = 12

(ii) No: of passengers who enjoyed only one of the 3 drinks } : 12+2+25 = 39

(iii) Exactly 2 of drinks = 15 + 5 + 15 = 45.

(iv) None of the drinks = $|\overline{T \cup W \cup N}| = |U| - |T \cup W \cup N|$

$= 120 - [52 + 62 + 75 - 35 - 35 - 35 + 20] = 120 - 104 = 16$

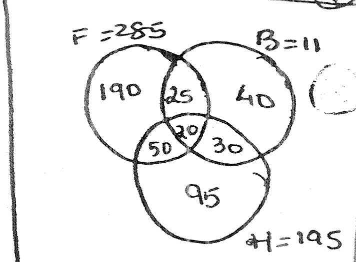
(10) A survey of 500 television watches produced the following information. 285 watch football games, 195 watch Hockey games, 115 watch basket ball games, 45 watch football & basket ball games, 70 watch football & hockey games, 50 watch Hockey and basket ball games and 50 do not watch any of the three kinds of games

(a) How many people in the survey watch all three kinds of games.

(b) How many people watch exactly one of the sports

Sol: $|U|=500$, $|F|=285$, $|H|=195$, $|B|=115$, $|F\cap B|=45$

$|F\cap H|=70$, $|H\cap B|=50$, $|\overline{F\cup B\cup H}|=50$



(a) To find: No: of people's watching 3 kinds of game $\}$: $|F\cap B\cap H|=?$

$|F\cup B\cup H| = |U| - |F\cap B\cap H| = 500-50 = 450.$

$|F\cup B\cup H| = |F|+|B|+|H| - |F\cap B| - |B\cap H| - |F\cap H| + |F\cap B\cap H|$

$450 = 285+115+195-45-70-50+|F\cap B\cap H|$

$\therefore |F\cap B\cap H| = 20.$

$\therefore$ 20 people watch all the three.

(b) To FIND: EXACTLY ONE OF THE SPORTS

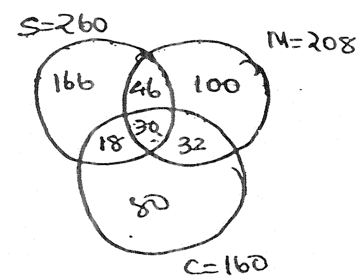
No: of people watch exactly one of the sports $\}$ $190+40+95 = 325.$

(11). In a survey of students at Florida state university the following information was obtained. 260 were taking a statistics course, 208 were taking a mathematics course, 160 were taking a Computer programing course 176 were taking statistics & mathematics, 48 were taking statistics and computer programing, 62 were taking mathematics courses and computer programming, 30 were taking 3 kinds of courses & 150 were taking none of the 3 course.

Sol: $|S|=260$, $|M|=208$, $|C|=160$, $|S\cap M|=76$ $\quad |U|=?$

$|S\cap C|=48$, $|M\cap C|=62$, $|S\cap M\cap C|=30$

$|\overline{S\cup M\cup C}|=150.$



(a) How many students were surveyed.

Sol: The total no: of students surveyed

$\therefore |U| = |S\cup M\cup C| + |\overline{S\cup M\cup C}|$

$= |S|+|M|+|C| - |S\cap M| - |S\cap C| - |M\cap C| + |S\cap M\cap C| + |\overline{S\cup M\cup C}|$

$= 260+208+160-76-62-48+30+150 = 622$

b) How many students were taking a statistics & mathematics course but not a computer programing course.

Sol) $S\cap M\cap \overline{C} = 46$

c) How many were taking a statistics and a computer course but not a mathematics course?

Sol $(S\cap C\cap \overline{M}) = 18$

d) How many were taking a Computer programming a mathematics course but not a statistics course.

Sol: $(C\cap M\cap \overline{S}) = 32$

(e) How many were taking a statistics but not taking a course in mathematics or in computer program.

Sol: $(S \cap \overline{M} \cap \overline{C}) = 166$

(f) How many were taking computer programming course but not taking a statistics course or maths.

Sol: $C(\cap \overline{S} \cap \overline{M}) = 80$.

(g) How many were taking a mathematics course but not statistics course or a computer programming course.

Sol: $(M \cap \overline{S} \cap \overline{C}) = 100$

(12) Of 30 personal computer (PC) owned by faculty members in a certain university department, 20 run windows, 8 have 21 inch monitors, 25 have CD ROM drivers, 20 have atleast two of these features and six have all three.

(a) How many PCs have atleast one of these features

(b) How many have exactly one features.

(c) How many have none of these features.

(d) How many faculty have computers with exactly 2 of these features.

Sol: Gn: $|W| = 20$, $|M| = 8$, $|C| = 25$, $|W \cap M \cap C| = 6$.

$|(W \cap M) \cup (W \cap C) \cup (M \cap C)| = 20$.

$|(W \cap M| + |W \cap C| + |M \cap C| - |(M \cap C) \cap (W \cap C)| - |(W \cap M) \cap (M \cap C)| -$

$|(W \cap C) \cap (M \cap C)| + |(W \cap M) \cap (W \cap C) \cap (M \cap C)| = 20$.

$\therefore |W \cap M| + |W \cap C| + |M \cap C| - 2|M \cap W \cap C| = 20$

$\therefore |W \cap M| + |M \cap C| + |M \cap C| = 20 + (2 \times 6) = 32$.

(a) Total no: of PC's with atleast one feature

$|M \cup W \cup C| = |M| + |W| + |C| - |M \cap W| - |M \cap C| - |W \cap C| + |W \cap M \cap C|$.

$= 20 + 8 + 25 - 36 + 6 = 27$

(b) $|\overline{(M \cup W \cup C)}| = |U| - |M \cup W \cup C| = 30 - 27 = 3$.

(c) Exactly one feature: The no: with atleast 1 feature - The no: with atleast 2 features.

$= 27 - 20 = 7$.

(d) Exactly two features = The no: atleast 2 features - The no: atleast 3 features

$= 20 - 6 = 14$.

$= 1000 - [333 + 200 + 142 + 48 - [66 + 47 + 15 + 28 + 6 + 9] + [9 + 3 + 2 + 1] + 0]$

$= 1000 - 564$

$\therefore |A \cup B \cup C \cup D| = 436.$

(17) Consider a set of integers from 1 to 250. Find how many of these no: are divisible by 5 or 6 or 8. Also indicate how many are divisible by 5 or 6 but not by 8 and divisible by 5 or 6

Sol: A: Set of integers divisible by 5.
B: Set of integers divisible by 6
C: Set of integers divisible by 8.

(a) TO FIND: No: of integers divisible by 5 or 6. $= |A \cup B| = ?$

$|A \cup B| = |A| + |B| - |A \cap B| \rightarrow (1)$

(b) TO FIND: No: of integers divisible by 5 or 6 or 8 $= |A \cup B \cup C| = ?$

(c) TO FIND: No: of integers divisible by 5 or 6 but not by 8
$= |A \cup B \cup C| - |C| = ?$

$|A| = \left[\frac{250}{L.C.M(5)}\right] = 50$; $|B| = \left[\frac{250}{L.C.M(6)}\right] = 41$; $|C| = \left[\frac{250}{L.C.M.(8)}\right] = 31.$

$|A \cap B| = \left[\frac{250}{LCM(5,6) = 30}\right] = 8$; $|A \cap C| = \left[\frac{250}{LCM(5,8) = 40}\right] = 6$

$|B \cap C| = \left[\frac{250}{LCM(6,8) = 24}\right] = 10$; $|A \cap B \cap C| = \left[\frac{250}{LCM(5,6,8) = 120}\right] = 2$

(1) $\Rightarrow |A \cup B| = |A| + |B| - |A \cap B|$
$= 50 + 41 - 8$
$= 83$

(b) $\Rightarrow |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$
$= 50 + 41 + 31 - 8 - 6 - 10 - 2$
$= 100.$

(c) $\Rightarrow |A \cup B \cup C| - |C| = 100 - 31 = 69.$

HW:

Find the number of integers between 1 & 1000 inclusive that are divisible by none of 5, 6 and 8.

⑬ How many positive integers not exceeding 500 are divisible by 7 or 11

Sol: A → Set of integers divisible by 7.

B : Set of integers divisible by 11.

$|A| = \left[\frac{500}{L.C.M(7)}\right] = 71$; $|B| = \left[\frac{500}{L.C.M(11)}\right] = 45$; $|A \cap B| = \left[\frac{500}{L.C.M(7,11)}\right] = 6$

The no: of integers divisible by 7 or 11 $= |A \cup B| = ?$

$|A \cup B| = |A| + |B| - |A \cap B| = 71 + 45 - 6 = 110$

---

⑭ Determine the no: of integers between 1 & 250 that are not divisible by 2, 3 & 5.

Sol: Let A: Set of integers b/n 1 & 250 are divisible by 2

B: Set of integers b/n 1 & 250 are divisible by 3

C: Set of integers b/n 1 & 250 are divisible by 5.

$\therefore |A| = \left[\frac{250}{L.C.M(2)}\right] = 125$; $|B| = \left[\frac{250}{L.C.M(3)}\right] = 83$; $|C| = \left[\frac{250}{L.C.M(5)}\right] = 50$

$|A \cap B| = \left[\frac{250}{L.C.M(2,3)}\right] = 41$; $|B \cap C| = \left[\frac{250}{LCM(3 \times 5)}\right] = 16$; $|A \cap C| = \left[\frac{250}{L.C.M(2,5)}\right] = 25$

$|A \cap B \cap C| = \left[\frac{250}{L.C.M(2,3,5)}\right] = 8$.

$\therefore |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

$= 125 + 83 + 50 - 41 - 16 - 25 + 8 = 184.$

No: of integers that are not divisible by 2, 3 or 5. $\Big\}$: $250 - 184 = 66$

---

⑮ Find the no: of integers b/n 1 & 1000 that are not divisible by any of 3, 5, 7 & 22.

Sol: Let A = Set of integers b/n 1 & 1000 are divisible by 3

B = Set of integers b/n 1 & 1000 are divisible by 5.

C = Set of integers b/n 1 & 1000 are divisible by 7.

D = Set of integers b/n 1 & 1000 are divisible by 22.

$|A| = \left[\frac{1000}{L.C.M(3)}\right] = 333$, $|B| = \left[\frac{1000}{L.C.M(5)}\right] = 200$, $|C| = \left[\frac{1000}{L.C.M(7)}\right] = 142$

$|D| = \left[\frac{100}{L.C.M(22)}\right] = 45$; $|A \cap B| = \left[\frac{1000}{L.C.M(3,5)}\right] = 66$; $|A \cap C| = \left[\frac{1000}{L.C.M(3,7)}\right] = 47$

$|A \cap D| = \left[\frac{1000}{L.C.M(3,22)}\right] = 15$; $|B \cap C| = \left[\frac{1000}{L.C.M(5,7)}\right] = 28$; $|C \cap D| = \left[\frac{1000}{L.C.M(7,22)}\right] = 6.$

$|B \cap D| = \left[\frac{1000}{L.C.M(5,22)}\right] = 9$, $|A \cap B \cap C \cap D| = \left[\frac{1000}{L.C.M(3,5,7,22)}\right] = [0.43] \approx 0.$

No: of integers not divisible by 3, 5, 7 & 22 $\Big\}$: $|\overline{A \cup B \cup C \cup D}| = ?$

$|\overline{A \cup B \cup C \cup D}| = 101 - |A \cup B \cup C \cup D| =$