Sentence :-

The Number of words Making a Complete gramatical structure, having sense and Meaningful is Called as a sentence

There are two types of Sentences :-

1. Declarative sentence
2. Non Declarative sentence

1. Declarative Sentence

A Declarative sentence Makes a Statement that declares something (or) give reliable information (or) Idea

Ex :- (i) $1+6=7$

(ii) Chennai is Capital of Tamilnadu

(iii) Newdelhi is in England

2. Non - Declarative sentence :-

Statement that does not declares Something (or) gives idea are Called non-declarative Sentences

Ex :- (i) Imperative sentence — Command, Wishes, Request

(ii) Exclamatory sentence [!] Express Strong feelings

(iii) Intragative sentence [?]

* Statement :- [valid or Invalid]

Statement is a Combination of sentence or sentences

Prepositional Statement :-

A preposition is a declarative statement which is eithea True or False but not both

Non - preposition :-

Sentences having command, exclamate, Questions and neithea True or false statements

Truth Value :-

the Truth (or) Falist of a preposition is called Truth value

Notation :-

If a preposition is True then its truth value is 'T' Similarly if a preposition is False the truth value is 'F'

Preposition :- Prepositions are Small lettea (or) captial lettea. It represented with Colon (:)

Examples of preposition with Truth values	Examples of Non-preposition Truth values
1. 5 is aprime Number : T	1. How old are you ?
2. $8 \div 4 = 2$: T	2. The peacock is very Beautiful !
3. $1+2=5$: F	3. Just obey my orders [Command]
	4. $x+3=7$ [Neithea True or false]

prepositional logics :-

The area of Logic that deals with prepositions is Called as preposition logics

Primary / Atomic Statements :-

A declarative sentence that cannot be further Splitted into Simple sentences is called as primary / Atomic sentences

the statement can't be divide called primary

Ex:- P : "Latha is a Teacher"

Note :- more than one statement is compound

Compound or Composite Statement

A Statement which Contains one or more primary statements along with some Connectives is Called (or) Composite Statement

Ex:- P : "Latha is a Teacher"

q : Latha teaches Java programming

Compound :-

Latha is a teacher and teaches Java programming

Connectives (or) logical operators :-

A Connectives is an operation which is used to Connect [more than two] statements

Ex:- 1. AND [Conjunction] $\wedge$

2. OR [Disjunction] $\vee$

3. NOT [Negation] $\sim / \neg$

Statement formula :-

A statement formula is an expression which is a string consisting of variables connectives and paranthesis

Truth Table :-

A Truth table displays the relationship between the truth values of the preposition

Logical Connectives :-

4. IF --- THEN (Conditional) $\longrightarrow$

5. IFF (If and only If) $\longleftrightarrow$ Bi Conditional

1. NOT [NEGATION]

Symbol :- $\sim / \neg$

P	$\neg P$
T	F
F	T

Ex:-1 $p : 2+5 > 1$ (T)

$\sim p : 2+5 \leq 1$ (F)

Ex:-2 p: It is hot today (T)

$\sim$p: It is NOT hot today (F)

2. AND [CONJUNCTION]

Symbol :- $\wedge$

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex:-1 p : It is raining today (T)

q : I am having Cold (T)

$p \wedge q$: It is raining today and I am having Cold (T)

2:- $p : 3 < 5$ (T)

$q : 2+3 = 6$ (F)

$p \wedge q : 3 < 5 \wedge 2+3 = 6$ (F)

3. $p : 3 < 5$ (T)

$q : 3 * 2 = 6$ (T)

$p \wedge q : 3 < 5 \wedge 3 * 2 = 6$ (T)

3. OR [DISJUNCTION] :-

Symbol :- $\vee$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Ex:-1

$p : 3 < 5$ (T)

$q : 2+3 = 6$ (F)

$p \vee q : 3 < 5 \vee 2+3 = 6$ (T)

2. p : 3 is a positive integer (T)

q : $\sqrt{3}$ is a Rational number (T)

$p \vee q$: 3 is a positive integer OR $\sqrt{3}$ is a Rational number (T)

4. IF ---- THEN [CONDITIONAL]

Symbol :- $\longrightarrow$

p	q	$p \longrightarrow q$	$q \longrightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Ex :- p : If I don't get Money (T)

q : I Shall buy a Car (T)

$p \rightarrow q$: If I don't get money then I shall by a Car (T)

2. $P : 3 + 6 = 10$ (F)

$Q : 2 + 6 = 9$ (F)

$P \rightarrow q$: If $3+6 = 10$ then $2+6 = 9$ (T)

5. IFF [BICONDITIONAL] If and only Iff :-

Symbol :- $\longleftrightarrow$ (or) $\rightleftarrows$

P	q	$P \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Ex:- p : you Can Take a flight (T)

q : If you buy a ticket (T)

$p \leftrightarrow q$: you Can Take a flight If and only If you buy a ticket (T)

The prepositions $p \rightarrow q$ and $p \leftrightarrow q$ are usually represented as Arrows :-

$P \rightarrow q$	$P \leftrightarrow q$
1. If p then q	1. p if and only if q
2. P implies q	2. p iff q
3. p only if q	3. p is necessary and sufficient for q
4. q is necessary for p	4. If p then q and If q then p
5. q whenever P [$(\neg q \vee p)$]	
6. q is implied by P	
7. q is sufficient for p	

Problems :-

Symbolise the following Statements using logical Connectives :-

Statement :-

1. If either Ram takes C^{++} OR Kumar takes Java then Latha will take Python as a subject

A. P : Ram takes C^{++}

Q : Kumar takes Java

R : Latha takes Python

$(P \vee Q) \longrightarrow R$

2. If the Moon is out AND it is not snowing THEN Neeraja goes out for a walk

P : The Moon is out

Q : It is Snowing (or) It is not snowing

R : Neeraja goes out for a walk

$$(P \wedge Q) \rightarrow R \quad (\text{or}) \quad (P \wedge \sim Q) \rightarrow R$$

3. IF it is NOT the Case that tharun goes out for a walk, IFF it is NOT snowing OR the moon is out

P : The moon is out

Q : It is Snowing

R : Tharun goes out for a walk

$$\neg [R \leftrightarrow (\neg q \vee p)]$$

4. Arun Can access the Internet with in the Campus only, If he is a Computer Science major OR he is NOT a fresher to the Campus

P : Arun Can access the internet with in the Campus

Q : he is a Computer Science major

R : he is a fresher to the Campus

$$[P \rightarrow (Q \vee \sim R)]$$

Express the logical Connectives as an english sentence

p: Swimming at the beach shore is allowed

q: Sharks has been Spotted near the shore

(i) $\sim q$: Sharks has NOT been spotted near the shore

(ii) $p \wedge q$:

Swimming at the beach shore is allowed AND Sharks has been spotted near the shore.

(iii) $\sim p \vee q$:

Swimming at the beach shore is NOT allowed OR Sharks has been spotted near the shore

(iv) $p \rightarrow q$:

If Swimming at the beach shore is allowed THEN Sharks has been spotted near the shore

(v) $\neg q \rightarrow p$:

If Shaks has NOT been spotted near the shore THEN swimming at the beach shore is allowed.

(vi) $p \leftrightarrow \neg q$:

Swimming at the beach shore is allowed iff sharks has NOT been spotted near the shore

Converse, Inverse, Contraposition for a given statement :-

If $p \rightarrow q$ is a Conditional Statement THEN

Converse of $p \rightarrow q$	$q \rightarrow p$
Inverse of $p \rightarrow q$	$\neg p \rightarrow \neg q$
Contrapositive of $p \rightarrow q$	$\neg q \rightarrow \neg q$

Conv

Examples :- 1

statement :-

The home Team wins whenever it is raining.

Modified sentence :-

If it is raining then the home team wins

Converse :- If the home team wins then it is Raining

Inverse :- If it is NOT raining THEN the home team does NOT wins

Contrapositive :- If the home team does NOT wins THEN it is NOT Raning

Example :- 2

statement :-

A positive Integer is a prime only If it has no divisors other than 1 and itself.

Con p: Positive Integer is Prime.

q: It has no divisor other than one and itself.

(i) Converse :- $p \rightarrow q \Rightarrow q \rightarrow p$

If the number has no divisors other than one and itself then that positive integer is a prime.

(ii) Contra positive :- $p \rightarrow q \Rightarrow \neg q \rightarrow \neg p$

If the number has divisor's other than one and itself then that positive integer is not a prime.

(iii) Inverse :- $p \rightarrow q \Rightarrow \neg p \rightarrow \neg q$

If a positive integer is not a prime then that number has divisor's other than one and itself

NOTE :-

The Total number of Rows needed for the Truth table is 2^n where 'n' is number of variables

Ex:- 1

Construct a Truth table for $(p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge q$	$p \wedge r$	① v ②
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

Construct the Truth table for $\neg(\neg P \vee \neg q)$

P	q	$\neg P$	$\neg q$	$\neg P \vee \neg q$	$\neg(\neg P \vee \neg q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

Construct the Truth table for $(P \wedge q) \vee (P \wedge \neg q)$

P	q	$\neg q$	$P \wedge q$	$P \wedge \neg q$	① v ②
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	F	F
F	F	T	F	F	F

Construct the Truth table for $(\neg p \wedge q) \leftrightarrow (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$\neg p \vee \neg q$	① $\leftrightarrow$ ②
T	T	F	F	F	F	T
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	F	T	F

Construct the Truth table $\neg p \wedge (\neg q \wedge r)$ $\quad 2^3 = 8$

p	q	r	$\neg p$	$\neg q$	$\neg q \wedge r$	① $\wedge$ ②
T	T	T	F	F	F	F
T	T	F	F	F	F	F
T	F	T	F	T	T	F
T	F	F	F	T	F	F
F	T	T	T	F	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

2^4

Construct the Truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	① $\leftrightarrow$ ②
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	F

p	q	r	s	p ⟷ q	r ⟷ s	① ⟷ ②
F	T	T	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Construct a Truth table $(\neg p \wedge (\neg q \wedge r)) \vee ((q \wedge r) \vee (p \vee r))$

p	q	r	¬p	¬q	(¬q ∧ r)	(p ∨ r)	(q ∧ r)	(¬p ∧ (¬q ∧ r))
T	T	T	F	F	F	T	T	F
T	T	F	F	F	F	T	F	F
T	F	T	F	T	T	T	F	F
T	F	F	F	T	F	T	F	F
F	T	T	T	F	F	T	T	F
F	T	F	T	F	F	F	F	F
F	F	T	T	T	T	T	F	T
F	F	F	T	T	F	F	F	F

((q ∧ r) ∨ (p ∨ r))	① ∨ ②
T	T
T	T
T	T
T	T
T	T
F	F
T	T
F	F

⇒ prepositional Equivalence (or) logical equivalence

Symbol :- $\Leftrightarrow$ / $\equiv$

logical Equivalence (or) Equivalence Rule	Law of Proposition
1. Idempotent Law	$p \wedge p \equiv p$ $p \vee p \equiv p$
2. Commutative law	$p \wedge q \Leftrightarrow q \wedge p$ $p \vee q \Leftrightarrow q \vee p$
3. Associative law	Three terms having same symbol $p \wedge (p \wedge r) \Leftrightarrow (p \wedge q) \wedge r$ $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
4. Distributive law	Where different symbol $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
5. Complement Law	$p \wedge \neg p \Leftrightarrow F$ $p \vee \neg p \Leftrightarrow T$
6. De-Morgan's law	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
7. Domination law	$p \wedge F = F$ $p \vee T = T$
8. Identity law :	$p \wedge T = p$ $p \vee F = p$

9.	Absorption law	$p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$
10	Double Negation law	$\neg(\neg p) \Leftrightarrow p$
11.	Contrapositive law	$p \to q \Leftrightarrow \neg q \to \neg p$
12.	Condition as disjunction	$p \to q \Leftrightarrow \neg p \vee q$
13.	Bi Conditional as Conditionality Law	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ $(p \wedge q) \vee (\neg p \wedge \neg q)$
14	Exporation Rule (or) Law	$p \to (q \to r) \equiv (p \wedge q) \to r$
15.	Equivalence Rule	

Tautology, Contradiction, Contigency

1. Tautology :-

A statement or formula or Resultant column for which the Truth values are always "True" is called as Tautology

Ex :- Using Truth Table show that $[p \wedge (p \to q)] \to q$ is a Tautology.

p	q	$p \to q$	$[p \wedge (p \to q)] \to q$	$p \wedge (p \to q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	F
F	F	T	T	F

is a Tautology

2. Contradication :-

A statement or a formula or Resultant column for which the Truth values are Always "False" is called Contradication.

Ex:- using Truth table show that $(\neg P \wedge P) \wedge q$ is a contradication

P	q	$\neg P$	$\neg P \wedge P$	$(\neg P \wedge P) \wedge q$
T	T	F	F	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

is a Contradication

3. Contigency :-

A statement or formula or Resultant Column for which the Truth values are either True and False [Combination of True & False] is called as contigency.

Ex:- using Truth table show that $(P \vee q) \rightarrow (P \rightarrow q)$ is a Contigency.

P	q	$P \rightarrow q$	$P \vee q$	① → ②
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

is a Contigency

* verify a given statement is Tautology or not

1. $((p \vee q) \wedge \neg p) \to q$

2. $(\neg p \wedge (p \to q)) \to \neg q$

3. $(\neg q \wedge (p \to q)) \to \neg p$

p	q	$\neg p$	$p \vee q$	$(p \vee q) \wedge \neg p$	① $((p \vee q) \wedge \neg p) \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

is a Tautology

p	q	$\neg p$	$\neg q$	$(p \to q)$	$\neg p \wedge (p \to q)$	① $\to \neg q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

It is not Tautology

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \wedge (p \to q)$ ①	① $\to \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

It is Tautology

Verify the given Compound proposition is a tautology or not by using equivalence rules

① $q \vee (p \vee \neg q) \vee (\neg p \wedge \neg q)$ is a tautology

② $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$ is a tautology

1. $q \vee (p \vee \neg q) \vee (\neg p \wedge \neg q)$

given proposition is

$\Rightarrow$ ~~$(q \vee \neg q) \vee p$~~

$p \vee (Q \vee R) \Leftrightarrow (p \vee Q) \vee (p \vee R)$

$p \vee (Q \vee R) \Leftrightarrow (p \vee Q) \vee R$

$q \vee (p \vee \neg q) \vee (\neg p \wedge \neg q)$

$\Rightarrow (q \vee \neg q) \vee p \vee (\neg p \wedge \neg q)$ (Associative law)

$\Rightarrow (T \vee p) \vee (\neg p \wedge \neg q)$ (By apply Complement law)

$\Rightarrow T \vee (\neg p \wedge \neg q)$ (By applying Dominence law)

$\Rightarrow T \vee$ any statement $\Rightarrow$ True always

$\Rightarrow \boxed{T}$ (As per Dominence law)

$\therefore q \vee (p \vee \neg q) \vee (\neg p \wedge \neg q)$ is a tautology

2: $[(p \vee q) \wedge (\neg p \vee r)] \longrightarrow (q \vee r)$

Given proposition is

$\underbrace{[(p \vee q) \wedge [\neg p \vee r)]}_{P} \rightarrow \underbrace{(q \vee r)}_{q}$

$\neg [(p \vee q) \wedge (\neg p \vee r)] \vee (q \vee r)$ [Condition as disjunction

$[(\neg q \wedge p) \vee (p \wedge \neg r)] \vee (q \vee r)$ (Dominans Law) A applying negation

2. $[(p\vee q)\wedge(\neg p\vee r)]\rightarrow(q\vee r)$

Given $[(p\vee q)\wedge(\neg p\vee r)]\rightarrow(q\vee r)$

$\neg[(p\vee q)\wedge(\neg p\vee r)]\vee(q\vee r)$ [condition as disjunction]

Apply negation

$[(\neg p\wedge\neg q)\vee(p\wedge\neg r)]\vee(q\vee r)$ [Dominan's Law]

$(\neg p\wedge\neg q)\vee[(p\wedge\neg r)\vee(q\vee r)]$ (Associative law)

$(\neg p\wedge\neg q)\vee[(q\vee r)\vee(p\wedge\neg r)]$ [Commitative]

$(\neg p\wedge\neg q)\vee(q\vee r\vee p)\wedge(q\vee r\vee\neg r)]$ (Distributive)

$(\neg p\wedge\neg q)\vee(q\vee r\vee p)\wedge(q\vee T)$ (Complement law)

$(\neg p\wedge\neg q)\vee\underbrace{(q\vee r\vee p)\wedge T}_{(p\vee q\vee r)}$ (complement law)

$p\wedge T\equiv p$ Identity law

$(\neg p\wedge\neg q)\vee(p\vee q)\vee r$ (Associative law)

$\neg(p\vee q)\vee(p\vee q)\vee r$ (De-morgan's law)

(Compliment law)

$T\vee r$ by Dominance law.

T

$\therefore[(p\vee q)\wedge(\sim p\vee r)]\rightarrow(q\vee r)$ is a tautology

3. $(p\wedge q)\rightarrow(p\vee q)$

$p\rightarrow q\equiv\neg p\vee q$

$\neg(p\wedge q)\vee(p\vee q)$ (condition as disjunction)

$(\neg p\vee\neg q)\vee(p\vee q)$ (complement law)

$(\neg p\vee p)\vee(\neg q\vee q)$

$T\vee T$

T

$\therefore(p\wedge q)\rightarrow(p\vee q)$ is tautology

4. Verify $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology or Not

Sol

$(\neg q \wedge (p \to q)) \to \neg p$ is the given statement

$\underbrace{\hphantom{\neg q \wedge}}_{\text{Assume } p}$ $\underbrace{\hphantom{\neg p}}_{\text{Assume } q}$

Then by Condition as disjunction we can write

$p \to q$ as $\neg p \vee q$

$\Rightarrow \neg(\neg q \wedge (p \to q)) \vee \neg p$ By Condition of disjunction

$\Rightarrow (\neg(\neg q) \vee \neg(p \to q)) \vee \neg p$

$\Rightarrow q \vee \neg(p \to q) \vee \neg p$ By Dauble negation law

$\Rightarrow q \vee \neg(\neg p \vee q) \vee \neg p$ By Condition of disjunction

$\Rightarrow q \vee \neg(\neg p) \wedge \neg q \vee \neg p$

$\Rightarrow q \vee (p \wedge \neg q) \vee \neg p$ By Demorgon's law

$\Rightarrow (q \vee \neg q) \wedge (q \vee p) \vee \neg p$ By Distributive law

$\Rightarrow T \wedge (q \vee p) \vee \neg p$ By Complement law

$\Rightarrow T \wedge (q \vee (p \vee \neg p)$ By Associative law

$\Rightarrow T \wedge (q \vee T)$ By Complement law

$\Rightarrow T \wedge T$ By Dominant law

$\Rightarrow T$

5. verify $[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$ is a tautology or not ?

Given

$$[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$$

$$\Rightarrow [(p \vee q) \wedge \neg(\neg p \wedge \neg(q \wedge r))] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$$

$$\Rightarrow [(p \vee q) \wedge \neg(\neg(p \vee (q \wedge r)))] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$$

Double negation law

$$\Rightarrow [(p \vee q) \wedge (p \vee (q \wedge r))] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$$

By distributive law $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$$\Rightarrow [(p \vee q) \wedge (p \vee q) \wedge (p \vee r)] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$$

By idempotent law $p \wedge p = p$

$$\Rightarrow [(p \vee q) \wedge (p \vee r)] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$$

By De-morgan's law

$$\Rightarrow [\underbrace{(p \vee q) \wedge (p \vee r)}_{p}] \vee [\neg(p \vee q) \vee \neg(p \vee r)]$$

By complement law $p \vee \neg p \Leftrightarrow T$

$$\Rightarrow T$$

$$\Rightarrow [(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$$

is a Tautology.

Duality principle :-

A Compound statement in which "TRUE" is replace by "FALSE" and "FALSE" is replace by "TRUE", And also Conjunction ($\wedge$) is replaced by Disjunction ($\vee$) (or) Disjunction ($\vee$) replaced by Conjunction ($\wedge$) repectively this princple is Called Duality principle

Example :-

1. write the duality of $(P \wedge \neg q) \vee T$

The duality of $(P \wedge \neg q) \vee T = (P \vee \neg q) \wedge F$

2. $(P \vee F) \wedge (Q \wedge T)$

$(P \wedge T) \vee (Q \vee F)$ is the duality for above example

Logical equivalances Can be verified by using the following properties :-

1. p and q have the Same truth value after resolving the problem
2. $P \rightleftarrows q$ is a tautology (T)
3. In a given expression if we take LHS and RHS and derived either $L \cdot H \cdot S = R \cdot H \cdot S$ (or) $R \cdot H \cdot S = L \cdot H \cdot S$
4. Assume 'P' and derived 'Q' or assume 'Q' and derived 'P'

Ex:- Show the $p \rightarrow q \equiv \neg p \vee q$ (or) $p \rightarrow q \Leftrightarrow \neg p \vee q$

By using Truth table

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T
		①		②

Hence $p \rightarrow q$ logically equivalance to $\neg p \vee q$

$$[\, p \rightarrow q \equiv \neg p \vee q \,]$$

By using Truth table show that the following statements are logically equivalent to each other

1. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Since, Truth values are same hence they are logically equivalent

2. $(p \rightarrow q) \wedge (p \wedge q) \cancel{\vee (\neg p \wedge \neg q)} \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	F	T	T

2. $(p \to q) \wedge (q \to p) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

p	q	$p \to q$ ①	$q \to p$ ②	① ∧ ②	¬p	¬q	$p \wedge q$ ③	$\neg p \wedge \neg q$ ④	③ ∨ ④
T	T	T	T	T	F	F	T	F	T
T	F	F	T	F	F	T	F	F	F
F	T	T	F	F	T	F	F	F	F
F	F	T	T	T	T	T	T	T	T

Since, Truth values are same ⟶ T F F F

Hence they are logically equivalent

Method :- 2

1. Show that $p \to q \Leftrightarrow \neg q \to \neg p$ is a Tautology

Given statement is

$\Rightarrow p \to q \Leftrightarrow \neg q \to \neg p$

We know that $\boxed{p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$\Rightarrow (p \to q) \to (\neg q \to \neg p) \wedge ((\neg q \to \neg p) \to (p \to q))$

$\Rightarrow$ w.k.T $\boxed{p \to q \equiv \neg p \vee q}$

$\Rightarrow \neg(p \to q) \vee (\neg q \to \neg p) \wedge \neg(\neg q \to \neg p) \vee (p \to q)$

$\Rightarrow \neg(\neg p \vee q) \vee (q \vee \neg p) \wedge \neg(q \vee \neg p) \vee (\neg p \vee q)$

$\Rightarrow (p \wedge \neg q) \vee (q \vee \neg p) \wedge (\neg q \wedge p) \vee (\neg p \vee q)$

Disbutive

$\Rightarrow (p \vee q \vee \neg p) \wedge (\neg q \vee q \vee \neg p) \wedge (\neg q \vee \neg p \vee q) \wedge (p \vee \neg p \vee q)$

$\Rightarrow (\underbrace{p \vee \neg p}_{T} \vee q) \wedge (\underbrace{q \vee \neg q}_{T} \vee \neg p) \wedge (\underbrace{q \vee \neg q}_{T} \vee \neg p) \wedge (\underbrace{p \vee \neg p}_{T} \vee q)$

$\Rightarrow (T \vee q) \wedge (T \vee \neg p) \wedge (T \vee \neg p) \wedge (T \vee q)$

$\Rightarrow T \wedge T \wedge T \wedge T$

$\Rightarrow T \wedge T$

$\Rightarrow T$

$\therefore (p \to q) \Leftrightarrow (\neg q \to \neg p)$ is a Tautology

2. Show that $\{[(p \vee \neg p) \to q] \to [(p \vee \neg p) \to r]\} \to (q \to r)$

is a tautology ?

Given

$\{[(p \vee \neg p) \to q] \to [(p \vee \neg p) \to r]\} \to (q \to r)$

complement law $p \vee \neg p \Leftrightarrow T$

$(T \to q) \to (T \to r) \to (q \to r)$

By condition as disjunction law

$(\neg T \vee q) \to (\neg T \vee r) \to (q \to r)$

$(F \vee q) \to (F \vee r) \to (q \to r)$

By Identity law

$(q \to r) \to (q \to r)$

By condition as disjunction

$(\neg q \vee r) \to (\neg q \vee r)$

Again condition as disjunction

$\neg(\neg q \vee r) \vee (\neg q \vee r) \Rightarrow (\neg q \vee r) \vee \neg(\neg q \vee r)$

by Commutative law

by Complement law

$\Rightarrow p \vee \neg p \Rightarrow T$

T

$\therefore \{[(p \vee \neg p) \to q] \to [(p \vee \neg p) \to r]\} \to (q \to r)$

is a tautology

3. Show that $(q \to (p \wedge \neg p)) \to (r \to (p \wedge \neg p)) \to (r \to q)$

Given $(q \to (p \wedge \neg p)) \to (r \to (p \wedge \neg p)) \to (r \to q)$

complement law $p \wedge \neg p \Leftrightarrow F$

$\Rightarrow (q \to F) \to (r \to F) \to (r \to q)$

we know that $p \to q = \neg p \vee q$

$\Rightarrow (\neg q \vee F) \to (\neg r \vee F) \to (\neg r \vee q)$

Apply distributive law

$(\neg q \to \neg r) \vee F \to (\neg r \vee q)$ Identity law $p \vee F = p$

$(\neg q \to \neg r) \to (\neg r \vee q)$

$p \to q \equiv \sim p \vee q$

$\neg(\neg q) \vee \neg r \to (\neg r \vee q)$

$q \vee \neg r \to (\neg r \vee q)$

$q \vee \neg r \to (q \vee \neg r)$

$\neg \underline{(q \vee \neg r)} \vee (\underbrace{q \vee \neg r}_{p})$ $[\because p \vee \neg p \equiv T]$

T

$\therefore$ the given statement is a tautology

4. Show that $(p \to q) \wedge (r \to q) \Rightarrow (p \vee r) \to q$ is a Tautology

Given that

$(p \to q) \wedge (r \to q) \Rightarrow (p \vee r) \to q$

Condition as disjunction

$(\neg p \vee q) \wedge (\neg r \vee q) \Rightarrow (\neg p \vee \neg r) \vee q$

Distributive law

$(\underbrace{\neg p \wedge \neg r}_{T}) \vee q \Rightarrow (\underbrace{\neg p \vee \neg r}_{T}) \vee q$

$\therefore p \rightarrow \equiv T$

T

5\. Show that $p \Rightarrow p \rightarrow q$

Given that $p \Rightarrow p \rightarrow q$

Condition as disjunction

$p \Rightarrow \neg p \vee q$

$\neg p \vee (\neg p \vee q) \Rightarrow (\underbrace{\neg p \vee \neg p}_{T}) \vee q$

(Complement law) (By Associative law)

$T \vee q$

T

L.H.S = R.H.S :-

6\. Show that $\underset{①}{p \rightarrow (q \vee r)} \Leftrightarrow \underset{②}{\neg r \rightarrow (p \rightarrow q)} \Leftrightarrow \underset{③}{(p \wedge \neg q) \rightarrow r}$

L.H.S :-

$p \rightarrow (q \vee r)$

$= \neg p \vee (q \vee r)$ Condition as disjunction

$\Rightarrow (q \vee r) \vee \neg p$ Commutative law

$\Rightarrow (r \vee q) \vee \neg p$

$\Rightarrow r \vee (q \vee \neg p)$ By Associative law

$\Rightarrow r \vee (\neg p \vee q)$ Commutative law

Assume

$\Rightarrow \neg r \rightarrow (p \rightarrow q)$ [$p \rightarrow q \equiv \neg p \vee q$ Condition as disjunction]

R.H.S

Consider a statement ②

L.H.S $\neg r \rightarrow (p \rightarrow q)$

$\Rightarrow (p \rightarrow q) \rightarrow \neg r$

$\Rightarrow \neg (p \rightarrow q) \rightarrow \neg(\neg r)$ [$\because p \rightarrow q \equiv \neg q \rightarrow \neg p$]

$\Rightarrow \neg (\neg p \vee q) \rightarrow \neg \neg r$

$\Rightarrow \neg(\neg p \vee q) \rightarrow r$ $[\because \neg\neg(p) = p]$

$\Rightarrow \neg(\neg p) \wedge \neg q \rightarrow r$

$\Rightarrow p \wedge \neg q \rightarrow r$

R.H.S

7 show that $(\neg p \wedge (\neg q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r)) \Leftrightarrow r$

Given that

$(\neg p \wedge (\neg q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$

Associative law Distributive law

$(\neg p \wedge \neg q) \wedge r \vee (q \vee p) \wedge r$

Commutative law

$\neg(p \vee q) \wedge r \vee (p \vee q) \wedge r$

$[\neg(p \vee q) \vee (p \vee q)] \wedge r$ $[\because p \vee \neg p = T]$

Complement

$T \wedge r$

r

R.H.S

8. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

R.H.S

$p \rightarrow (q \wedge r)$ $[p \rightarrow q \Leftrightarrow \neg p \vee q)$

$\neg p \vee (q \wedge r)$

$(\neg p \vee q) \wedge (\neg p \vee r)$ [Distributive

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$(p \rightarrow q) \wedge (p \rightarrow r)$

L.H.S

(or)

Take L.H.S $(p \rightarrow q) \wedge (p \rightarrow r)$

$(\neg p \vee q) \wedge (\neg p \vee r)$

$\neg p \vee (q \wedge r)$

$p \rightarrow (q \wedge r)$ R.H.S

Show that $p \rightarrow (q \rightarrow p) \equiv \neg p \rightarrow (p \rightarrow q)$

L.H.S — R.H.S

$p \rightarrow (q \rightarrow p)$ — $\neg p \rightarrow (p \rightarrow q)$

$\neg p \vee (q \rightarrow p)$ — $\neg(\neg p) \vee (p \rightarrow q)$

$\neg p \vee (\neg q \vee p)$ — $\neg(\neg p) \vee (\neg p \vee q)$

$(\neg p \vee p) \vee \neg q$ — $(p \vee \neg p) \vee q$ [Association]

$T \vee \neg q$ — $T \vee q$

T

L.H.S = R.H.S

Show that $\neg\{(\neg p \wedge q) \vee (\neg p \wedge \neg q)\} \vee (p \wedge q) \equiv p$

the duality of above statement

$\Rightarrow \neg[(\neg p \vee q) \wedge (\neg p \vee \neg q)] \wedge (p \vee q) \equiv p$

$\Rightarrow [(\neg(\neg p) \wedge \neg q) \vee (\neg(\neg p) \wedge \neg(\neg q))] \wedge (p \vee q)$

$\Rightarrow [(p \wedge \neg q) \vee (p \wedge q)] \wedge (p \vee q)$

$\Rightarrow [p \wedge (q \vee \neg q)] \wedge (p \vee q)$ [Complement law, $p \vee \neg p \equiv T$]

$\Rightarrow [p \wedge T] \wedge (p \vee q)$ [Identity law $p \wedge T = p$]

$\Rightarrow p \wedge (p \vee q)$ [Absorption law $p \wedge (p \vee q) \equiv p$]

$\Rightarrow p$

Rules of Inference :

To prove this kind of theorems we consider

1. Hypothesis / premesis
2. Conclusion

1. Hypothesis / premesis :-

Hypothesis is nothing but the given premesis which are assumed to be true always

Conclusion is nothing but the result deriverd from the Hypothesis which is also true

Name of the rule	Formula	premesis	Conclusion
1. MODUNES PONESE	$p \to q$ is a statement Assume p is True Dervie q is True $\frac{p \to q}{p}$	$p \to q$, p	q
2. MODUNES of Tollerance	$p \to q = \neg q$ $\frac{p \to q \quad \neg q}{\therefore \neg p}$	$p \to q$, $\neg q$	$\neg p$
3. Addition rule	(i) $\frac{p}{p \vee q}$ (ii) $\frac{q}{p \vee r}$	p, q	$p \vee q$
4 Conjunction Rule	$\frac{p, q}{p \wedge q}$	p, q	$p \wedge q$
5. Disjunction	(i) $\frac{p \vee q, \sim q}{p}$ (ii) $\frac{p \vee q, \sim p}{q}$	$p \vee q, \sim q$ $p \vee q, \sim p$	p, q
6. SIMPIIFICATION	(i) $\frac{p \wedge q}{p}$ (ii) $\frac{p \wedge q}{q}$	$p \wedge q$	p, q

7 HYPOTHETICAL SYLLOGISM	$p \rightarrow q$ and $q \rightarrow r$ / $\overline{}$ $p \rightarrow r$	$p \rightarrow q$, $q \rightarrow r$	$p \rightarrow r$
8. RESOLUTION	$p \vee q$ and $\sim p \vee r$; $\sim p \vee r$ / $q \vee r$ $\therefore q \vee r$	$p \vee q$, $\sim p \vee r$	$q \vee r$
9. DILEMMA	$p \vee q$, $p \rightarrow r$, $q \rightarrow r$ / r	$p \vee q$, $p \rightarrow r$, $q \rightarrow r$	r

proofs :-

Direct proof :-

when a conclusion is derived from a set of premises by using equivalence rules & implication rules then this derivation is called as direct proof

Ex :- 1

Show that $\sim p$ is a valid conclusion from the premises $\sim p \vee q$, $\sim(q \vee r)$ & $\sim r$ by using logical implication

Step	PREMISES	REASON
1	$\sim(q \vee r)$	Rule P
2	$\sim q \wedge \sim r$	1. Rule T, Demorgans law
3.	$\sim q$	2. Rule T, Simplification Law
4.	$\sim p \vee q$	Rule P
5.	$\sim p$	{3, 4}, Rule T : Disjunction Syllogism

$\therefore \sim p$ is a valid conclusion for the given premises

2. Show that t is a valid conclusion from the premises $p \to q$, $q \to r$, $r \to s$, $\sim s$, $p \vee t$ by using logical implication

P $\to$ premises

T $\to$ Transpose

STEP	PREMISE	REASON
1.	$p \to q$	Rule P
2.	$q \to r$	Rule P
3.	$p \to r$	{1, 2}, Rule T: Hypothetical syllogism
4.	$r \to s$	Rule P
5.	$p \to s$	{3, 4}, Rule T: Hypothetical syllogism
6.	$\sim s$	Rule P
7.	$\sim p$	{5, 6}, Rule T: Modunes of Tollerance
8.	$p \vee t$	Rule P
9.	t	{7, 8}, Rule T: Disjunction rule

$\therefore$ 't' is a valid conclusion from the given premises

3. Show that s is a valid conclusion from the premises $p \to q$, $p \to r$, $(s \vee p)$, $\sim(q \wedge r)$ by using logical implication.

Given premises :-

$p \to q$, $p \to r$, $(s \vee p)$, $\sim(q \wedge r)$

Conclusion :- s is a valid conclusion

step	premises	reason
1.	$\neg(q \wedge r)$	Rule P
2.	$\neg q \vee \neg r$	Rule T, Demorgans law
3.	$q \rightarrow \neg r$	Rule T, conditional law
4.	$p \rightarrow q$	Rule P
5.	$p \rightarrow \neg r$	{3, 4} Rule T Hypothetical Syllogism
6.	$p \rightarrow r$	Rule P
7.	$\neg r \rightarrow \neg p$	Rule T, contrapositive law
8.	$p \rightarrow \neg p$	{5, 7} Rule T, Hypothetical syllogsm
9.	$\neg p \vee \neg p$	Rule T, conditional as disjunction
10.	$\neg p$	{9} Rule T, Idempotent law
11.	$s \vee p$	Rule P
12.	s	{10, 11} Rule T. Disjunction law

$\therefore$ S is a Valid Conclusion for give premises

4. Show that 's' is a valid conclusion from the premises
$(p \rightarrow \neg q) \wedge (q \vee r) \wedge (\neg s \rightarrow p) \wedge \neg r$

Given premises are $(p \rightarrow \neg q), (q \vee r), (\neg s \rightarrow p), \neg r$

Conclusion are 's' is a valid conclusion.

Step	Premises	Reason
1.	$p \rightarrow \neg q$	Rule P
2.	$q \vee r$	Rule P
3.	$\neg q \rightarrow r$	{2} Rule T, Conditional as disjunction
4.	$p \rightarrow r$	{1, 3}, Rule T : Hypothetical syllogism
5.	$\neg r$ ~~[illegible]~~	Rule P
6.	$\neg p$	{4, 5}, Rule T : modunes of Tollerance
7.	$\neg s \rightarrow p$	Rule P
8.	$\neg(\neg s)$	{6, 7} Rule T : modunes of Tollerance
9.	s	{8} double negation

$p \rightarrow q \equiv \neg p \vee q$
$\equiv \neg(\neg p) \vee q$
$= p \vee q$

5. Show that $(p \vee q) \wedge (p \to r) \wedge (q \to s) \Rightarrow s \vee r$

Given premises $(p \vee q) \wedge (p \to r) \wedge (q \to s)$.

Conclusion are $s \vee r$

Step	premises	Reason
1.	$p \vee q$	Rule 'P'
2.	$\neg p \to q$	Rule 'T', Condition law
3.	$q \to s$	Rule 'P'
4.	$\neg p \to s$	{2,3}, Rule 'T', Hypothetical syllogism
5.	$\neg s \to p$	{4}, Rule 'T', Contrapositive law
6.	$p \to r$	Rule 'P'
7.	$\neg s \to r$	{5, 6} Rule 'T', Hypothetical Syllogism
8.	$s \vee r$	{7} Rule 'T', Condition law

$\therefore$ $s \vee r$ is a valid Conclusion for the given premises

6. Show that

$(p \to q) \wedge (r \to s)$, $(q \wedge m) \wedge (s \to n)$

$\neg(m \wedge n)$, $(p \to r) \Rightarrow \neg p$ is a valid argument

Given premises $(p \to q) \wedge (r \to s)$, $(q \wedge m) \wedge (s \to n)$

$\neg(m \wedge n)$, $(p \to r)$ and Conclusion $\neg p$

steps	premises	Reason
1.	$(p \to q) \wedge (r \to s)$	Rule P
2.	$p \to q$	{1} Rule T, simplification rule
3.	$r \to s$	{1} Rule T, simplification rule
4.	$(q \wedge m) \wedge (s \to n)$	Rule P
5.	$q \wedge m$	{4} Rule T, simplification rule
6.	$s \to n$	{4} Rule T, simplification rule
7.	$p \to r$	Rule P
8.	$p \to s$	{7, 3} Rule T, Hypothetical syllogism
9.	$p \to n$	{8, 6} Rule T, Hypothetical syllogism
10.	$\neg(m \wedge n)$	Rule P
11.	$\neg m \vee \neg n$	Rule T, Demorgan's law
12.	$\neg n \vee \neg m$	Rule T, Commutative law
13.	$n \to \neg m$	{12} Rule T, Conditional law
14.	$p \to \neg m$	{13, 9} Rule T, Hypothetical Syllogism
15.	$m \to \neg p$	{14} Rule T, Contrapositive law
16.	$\neg p$	{15} Rule T, simplification rule

$\therefore$ $\neg p$ is a valid conclusion for given premises

7 Show that $p \vee s$ is valid or invalid conclusion for the premises $(p \wedge q) \vee r$ and $r \rightarrow s \Rightarrow p \vee s$

Given premises $(p \wedge q) \vee r$ and $r \rightarrow s$ and conclusion $p \vee s$

Step	premises	Reason
1.	$(p \wedge q) \vee r$	Rule P
2.	$(p \vee r) \wedge (q \vee r)$	{1}, Rule T, Distributive law
3.	$p \vee r$	{2}, Rule T, simplification rule
4.	$q \vee r$	{2}, Rule T, simplification rule
5.	$r \rightarrow s$	Rule P
6.	$\neg r \vee s$	{5}, Rule T, Condition law
7.	$r \vee p$	{3}, Rule T, Commutative law
8	$p \vee s$	{6, 7}, Rule T Resolution Rule

$p \vee s$ are valid conclusion for given premises

8. If the labour market is perfect, then the wages of ~~the~~ all the employees (particular) are equal. But it is a case that wages for such persons are not equal. Therefore the labour market is not perfect. validate the above argument?

Given premises

Let p : Labour market is perfect

q : wages of ~~all~~ the employees are equal

Given premises :- $P \to q$, $\neg q$

Conclusion :- $\neg P$

Step	Premises	Reason
1.	$P \to q$	Rule P
2.	$\neg q$	Rule P
3.	$\neg P$	{1, 2} Rule T, modunes of Tollerance rule

$\therefore$ $\neg P$ is a valid Conclusion.

$\therefore$ The labour market is Not perfect.

9. If the music party Could not play music, or refereshemts or not delivered on time, then the christmas party would have been cancelled and the organizers would be angry if the party was Cancelled, then the refund would have to be made. No refund was made. Theore fore the music Party Could play the music.

sol:- Given premises : $(\neg P \vee \neg q) \to (r \wedge s)$, $r \to t$, $\neg t$

Conclusion : P

Steps	Premises	Reason
1.	$r \to t$	Rule P
2.	$\neg t$	Rule P
3.	$\neg r$	{1, 2}, Rule 'T', modunes of Tollerance
4.	$\neg r \vee \neg s$	{3}, Rule 'T', Addition Rule
5.	$\neg (r \wedge s)$	{4}, Rule 'T' Demorgan's Law

6.	$(\neg p \vee \neg q) \rightarrow (r \wedge s)$	Rule P
7.	$\neg(\neg p \vee \neg q) \vee (r \wedge s)$	{6}, Rule T Conditional law
7	$\neg p \vee \neg q$	{6}, Rule T Modunes ponese Rule
7.	$\neg(\neg p \vee \neg q)$	{6}, Rule T Modunes of Tolleriance $\dfrac{P \rightarrow q}{\therefore \neg P}$
9.	~~$\neg(\neg p \vee \neg q)$~~	
8.	$p \wedge q$	{8} Rule T, Double negation law
10.	p	{9} Rule T, simplification rule
8.	$\neg(\neg p)$	{7} Rule T Disjunction law
9		{8} Double negation law

Indirect Method :-

Indirect Method of solving rules of Inference problems.

for doing indirect method problems we intraduce the Negation for given Conclusion, as an Additional premises and then Solve the problem along with the given premises to derived the Conclusion problems :

1. using indirect method of proof show that

$p \rightarrow r,\ q \rightarrow s,\ p \vee q \Rightarrow s \vee r$

Given premises : $p \rightarrow r,\ q \rightarrow s,\ p \vee q$

Conclusion : $s \vee r$

Additional premises : $\neg(s \vee r)$ or $\neg s \wedge \neg r$

Steps	premises	Reason
1.	$\neg s \wedge \neg r$	Rule AP
2.	$\neg s$	{1}, Rule T Simplification rule
3.	$\neg r$	{1}, Rule T Simplification rule
4.	$p \rightarrow r$	Rule P
5.	$\neg p$	{3,4}, Rule T modunes of Tollerance
6.	$p \vee q$	Rule P
7.	q	{5, 6}, Rule T Disjunction rule
8.	$q \rightarrow s$	Rule P
9.	s	{8}, Rule T Modunes Ponese
10.	$s \wedge \neg s$	{2, 9}, Rule T Conjunction rule
11.	F	Rule T complement law

Here the result we obtain is not the orginal Conclusion [assume premises]

$\therefore$ the given Conclusion is valid for the given premises

2. Show that the given conclusion is valid by using indirect proof on the premises $p \to q$, $q \to r$, $p \vee r$, $\neg(p \wedge r)$ and conclusion r

Premises : $p \to q$, $q \to r$, $p \vee r$, $\neg(p \wedge r)$

Conclusion : r

Additional premises : $\neg r$

Step	Premises	Reason
1.	$\neg r$	Rule AP
2.	$p \vee r$	Rule P
3.	p	{1, 2} Rule T , Disjunction rule
4.	$p \to q$	Rule P
5.	q	{4} Rule T, modunes ponese
6.	$q \to r$	Rule P
7.	r	{6} Rule T, modunes ponese
8.	$\neg(p \wedge r)$	Rule P
9.	$\neg p \vee \neg r$	{8} Rule T, demorgan's law
10.	$p \to \neg r$	{9} Rule T, Conditional law
11.	$\neg p$	{10, 7} Rule T, modunes of Tollerance
12.	$p \wedge \neg p$	{3, 11} Rule T, Conjunction rule
	F	Rule T, Complement law

Here the result we obtained is not the original Conclusion (Additional premises)

∴ The given Conclusion is valid for the given premises

5. Check the validity of the Conclusion foor the given premises

$r \rightarrow \neg q$, $r \vee s$, $s \rightarrow \neg q$, $p \rightarrow q \Rightarrow \neg p$

premises : $r \rightarrow \neg q$, $r \vee s$, $s \rightarrow \neg q$, $p \rightarrow q$

Conclusion : $\neg p$

Additional premises : p

step	premises	Reason
1.	p	Rule 'AP'
2.	$p \rightarrow q$	Rule 'p'
3.	q	{1, 2}, Rule 'T', modunes ponese
4.	$r \rightarrow \neg q$	Rule 'p'
5.	$s \rightarrow \neg q$	Rule 'p'
6.	$(r \vee s) \rightarrow \neg q$	{4, 5}, Rule 'T', Equivalence Exploratory Rule
7.	$r \vee s$	Rule 'p'
8.	$\neg q$	{6, 7}, Rule 'T' modunes of Tollerance
9.	$q \wedge \neg q$	{3, 8}, Rule 'T', conjunction rule
10.	F	{9}, Rule 'T', complement law

Here the result we obtained is not the original conclusion (Addition premises)

∴ The given Conclusion is valid for the given Premises

* Conditional proof :-

If the Conclusion of the statement is of the form $p \rightarrow q$ then set an additional premises as p and Conclusion as q

Example :-

1. Show that $r \rightarrow s$ can be derived from $p \rightarrow (q \rightarrow s)$, $\neg r \vee p \Rightarrow q$

Given premises,

$p \rightarrow (q \rightarrow s)$, $\neg r \vee p$, q

and the Conclusion is s

Additional premises is r

[If the given premises is of the form $p \rightarrow q$ then additional premises 'p' and the Conclusion is 'q']

Step	premises	Reason
1.	r	Rule 'AP'
2.	$\neg r \vee p$	Rule 'p'
3.	p	{1, 2}, Rule 'T', Disjunction rule
4.	$p \rightarrow (q \rightarrow s)$	Rule 'p'
5.	$q \rightarrow s$	{4, 3}, Rule 'T', modunes of Tollerance
6.	q	Rule p
7.	s	{5, 6}, Rule 'T', modunes of Tollerance is the Conclusion

$\therefore$ $r \rightarrow s$ Can be derived from the given premises

2. Derive $p \rightarrow (q \rightarrow s)$ by using the premises

$p \rightarrow (q \rightarrow r)$, $q \rightarrow (r \rightarrow s)$

Given premises, $p \rightarrow (q \rightarrow r)$, $q \rightarrow (r \rightarrow s)$

and the Conclusion is $q \rightarrow s$

Additional premises is p

Step	Premises	Reason
1.	p	Rule 'AP'
2.	$p \rightarrow (q \rightarrow r)$	Rule 'P'
3.	$q \rightarrow r$	{1, 2}, Rule 'T', modunes of Tollerance ponese
4.	$q \rightarrow (r \rightarrow s)$	Rule 'P'
5.	$\neg q \vee (r \rightarrow s)$	{4}, Rule 'T', Conditional law
6.	$\neg q \vee (\neg r \vee s)$	{5}, Rule 'T', conditional law
7.	$\neg r \vee (\neg q \vee s)$	{6}, Rule 'T', Associative law
8.	$r \rightarrow (\neg q \vee s)$	{7}, Rule 'T', Conditional law
9.	$r \rightarrow (q \rightarrow s)$	{8}, Rule 'T', Conditional law
10.	$q \rightarrow (q \rightarrow s)$	{3, 9}, Rule 'T', Hypothetical syllogism
11.	$(q \wedge q) \rightarrow s$	{10}, Rule 'T', Exploratory rule
12.	$q \rightarrow s$	{11}, Rule 'T', Idempotent law

$\therefore$ $q \rightarrow s$ Can be derived from the given premises

3. Show that the below given premises or inconsistance

$p \to q, p \to r, q \to \neg r, p$

Given premises $p \to q, p \to r, q \to \neg r, p$

steps	premises	Reason
1.	$p \to q$	Rule 'P'
2.	$q \to \neg r$	Rule 'P'
3.	$p \to \neg r$	{1, 2}, Rule T, ~~modunes~~ Hypothetical Syllogism
4.	p	Rule 'P'
5.	$\neg r$	{3, 4}, Rule 'T', modunes ponese
6.	$p \to r$	Rule p
7.	$\neg p$	{5, 6}, Rule 'T', modunes of Tollerance
8.	$p \wedge \neg p$	{4, 7} Rule 'T', conjunction Rule
9.	F	{8}, Rule 'T', Complement law

∴ The given premises are inconsistance

4. Show that the following premises or inconsistance

$a \to (b \to c), d \to (b \wedge \neg c), (a \wedge d)$

Given premises are

$a \to (b \to c), d \to (b \wedge \neg c), (a \wedge d)$

Steps	premises	Reason
1.	$a \wedge d$	Rule 'p'
2.	a	{1}, Rule 'T', simplification rule
3.	d	{1}, Rule 'T', simplification rule
4.	$a \rightarrow (b \rightarrow c)$	Rule 'p'
5.	$b \rightarrow c$	{4}, Rule 'T', Modunes ponese
6.	$\neg b \vee c$	{5}, Rule 'T', conditional law
7.	$d \rightarrow (b \wedge \neg c)$	Rule 'p'
8.	$\neg(b \wedge \neg c) \rightarrow \neg d$	{7}, Rule 'T', contrapositive law of $p \rightarrow q \equiv \neg q \rightarrow \neg p$
9.	$\neg b \vee c \rightarrow \neg d$	{8}, Rule 'T', De morgan's law
10.	$\neg d$	{9}, Rule 'T', modunes ponese
11.	$d \wedge \neg d$	{3, 10}, Rule 'T', Conjunction Rule
	F	{11}, Rule 'T', Complement law

∴ the given set of premises are incansistance

5. Show that the following premises derived from the statements is inconsistance

Statement :- 1

If Jack misses many classes because of illness in ~~a~~ high school

Statement :- 2

If Jack fails in high school he is uneducated.

Statement :- 3

If Jack reads lot of books then he is not uneducated.

Statement :- 4

Jack misses many classes through illness and reads lots of books

~~Let~~ p : ~~If~~ Jack ~~misses~~ many classes ~~because~~

Sol Let

p : Jack misses many classes through illness

q : Jack fails in high school

r : Jack is uneducated

s : Jack reads lot of books.

premises are :-

$p \rightarrow q$, $q \rightarrow r$, $s \rightarrow \neg r$, $p \wedge s$

steps	premises	Reason
1.	$p \rightarrow q$	Rule 'P'
2.	$q \rightarrow r$	Rule 'P'
3.	$p \rightarrow r$	{1, 2}, Rule 'T', Hypothetical syllogism
4.	$s \rightarrow \neg r$	Rule 'P'
5.	$r \rightarrow \neg s$	{4}, Rule 'T', Contrapositive law
6.	$P \rightarrow \neg s$	{3, 5}, Rule 'T', Hypothetical syllogism
7.	$\neg P \vee \neg S$	{6}, Rule 'T', Conditional law
8.	$\neg(P \wedge S)$	{7}, Rule 'T', De morgans law
9.	$P \wedge S$	Rule 'P'
10	$P \wedge S \wedge \neg(P \wedge S)$ $\neg(P \wedge S) \wedge P \wedge S$	{8, 9}, Rule 'T', Conjunction Rule
	F	{10}, Rule 'T', Complement law

∴ The given premises are in Cosistance