Unit - 3

Counting :- Counting is the action of finding the number of elements of a finite set of objects

The Basics of Counting :-

$\boxed{\text{Fundamental principles of Counting}}$

Addition / sum Rule / principle of Principle Disjunctive Counting	Product / Multiplication Rule rule / principle of sequential rule
If an event occur in 'm' ways another event occur in 'n' ways and if these two events Cannot occur simultaneously Then one of the 2 event Can occur in m+n ways	If an event occurs in 'm' ways and another event Occur in 'n' ways then 2 events occur simultaneously in m·n ways

or

Task 1 $\boxed{}$ m ways $\xleftrightarrow[\text{Jobs}]{\text{No Common}}$ Task 1 $\boxed{}$ n ways

The total no. of ways in which either of the two task Can be done

and

Task 1 $\boxed{\text{m way}}$ following Task 2 $\boxed{\text{n ways}}$

PBS Based ON Sum Rule AND Product rule :-

1\. In how many ways Can we draw a heart or a shade from an ordinary deck of playing Cards? A heart or an Ace ? An ace or a king ? A Card numbered 2 through 10 ? A numbered Card or a king?

52 Cards.

♥ 13 ♠ 13

(a) No of heart = 13

No of shade = 13

SUM RULE = WAYS TO DRAW HEART (OR) SHADE

= 13 + 13

= 26 ways

(b) No of heart = 13

No of Ace = 3

♡	A 3 Ace
13	3

SUM RULE = WAYS TO DRAW HEART (OR) ACE

= 13 + 3

= 16 ways

(c) NO of Ace = 4

No of king = 4

SUM RULE = WAYS TO DRAW HEART OR KING

= 4 + 4

= 8 ways

(d) No of n 2 to 10 is 10

No of way a Card numbered 2 through 10

= 9 × 4

= 36 ways

(e) No of numbered Card = 36

No of king = 4

No of ways to draw Numbered card or a king

= 36 + 4

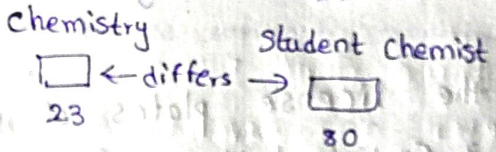
= 40 ways

2) Suppose that either a member of the chemistry faculty or a student who is a chemistry major is choosen as a representative to a university Commuitee How many different Choices are there for this representatal if there are 23 members of the chemisty faculty of 80 chemistry majors and non one is both a faculty member of a student.

Ans No: of ways to choose Chemistry faculty = 23 ways
No: of ways to choose a student who is a chemistry major = 80 ways

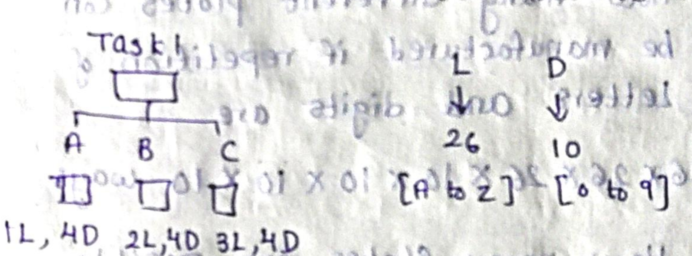
SUM RULE :- To choose a member of the chemistry faculty is never the same as choose a student who is a chemistry major because no one is both faculty member and a student.

POSSIBLE WAYS TO PICK THIS REPREDENTATIVES = 23 + 80
= 103 ways



3) How many different license plates are there that involve 1, 2 or 3 letter followed by 4 digits

Ans:-



1 Letter followed by 4 digits = $26 \times 10 \times 10 \times 10 \times 10 \rightarrow A$
2 Letter followed by 4 digits = $26 \times 26 \times 10 \times 10 \times 10 \times 10 \rightarrow B$
3 letter followed by 4 digits = $26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10 \rightarrow C$

SUM RULES :- No of different dicenese Plates = A+B+C
~~Product Rules~~

4) If a child can draw two kinds of faces and three kinds of hats, how many cartoons can she produce
Product

Ans
No. of ways face is drawn = 2 ways
No of ways hats is drawn = 3 ways

Product rule :- No of ways cartoon can be drawn = 2×3 = 6 ways

5) The chairs of an Auditorium are to be labeled with a Letter and a positive enteger not exceeding 100 what is the Largest no of chairs that Can be labelled differently

Task 1 Labelling chair

Product rule :-

No of ways that a chair Can be labelled $\Big\}$ = 26 X 100 ways

L → 26, I → 100

The Largest no of ways that a chair Can be labelled $\Big\}$ = 2600 ways

6. Suppose that the License plates of a certain state requre 3 English letters followed by 4 digits

(a) How many different plates Can be manufactured if repetition of Letters and digits are

Task 1

3L, 4D → 26, 10

Sol 26 X 26 X 26 X 10 X 10 X 10 X 10 ways

(b) How many States are possible if only the Letters Can be repeated

Sol 26 X 26 X 26 X 10 X 9 X 8 X 7

(c) only the digits Can be repeated

Sol 26 X 25 X 24 X 10 X 10 X 10 X 10 ways

(d) No repetitions are allowed at all

Sol 26 X 25 X 24 X 10 X 9 X 8 X 7

7) (i) How many 3 digits numbers can be formed using 1, 3, 4, 5, 6, 8 & 9

(ii) How many can be formed if no digits can be repeated.

Sol:- (i) No of ways each of 3 digit is filled using 1, 3, 4, 5, 6, 8, 9 } = 7 ways

100's 10's 1's
$\square$ $\square$ $\square$
7 7 7

∴ No of ways 3 digits number is can be formed = 7 × 7 × 7 ways

$\square$ $\square$ $\square$
7 6 5

(ii) No of ways 3 digit no is formed without repeatation } = 7 × 6 × 5 ways

→ 30 in value

8) (a) How many 2 digit or 3 digit numbers can be formed using the digits 1, 3, 4, 5, 6, 8 and 9 if no repetation is allowed.

sol No of ways each of 3 digit is filled using (1, 3, 4, 5, 6, 8, 9) } = 7 ways

2D
$\square$ $\square$
7 6

3D
$\square$ $\square$ $\square$
7 6 5

No of ways 2 digit no Can be formed without repetation } = 7 × 6 way = 42 ways

No of ways 3 digit no Can be formed without repetation } = 7 × 6 × 5 ways = 210 ways

No of ways 2 digit (or) 3 digit no Can be formed } = 42 + 210
= 252 ways

(b) How many numbers Can be formed using the digit 1, 3, 4, 5, 6, 8, 9 if no repetation are allowed

The number of digits are not specified so we Can form [without repetition]

No: of ways 1-digit is formed = 7 ways = A

No: of ways 2-digit is formed = 7×6 ways = B

No: of ways 3-digit is formed = $7 \times 6 \times 5$ ways = C

No: of ways 4-digit is formed = $7 \times 6 \times 5 \times 4$ ways = D

No: of ways 5-digit is formed = $7 \times 6 \times 5 \times 4 \times 3$ way = E

No: of ways 6-digit is formed = $7 \times 6 \times 5 \times 4 \times 3 \times 2$ = F

No: of ways 7-digit is formed = $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ = G

$\therefore$ No of way different no: Could be formed without repetition = A + B + C + D + E + F + G

9) How many three digit numbers are there which are even and have no repeated digits

Sol: for a no: to be even it must end with $\}$ = 0, 2, 4, 6 or 8

No: of ways each of the 3 digit is formed $\}$ = 10 ways (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Case (i)

Last Number = 0

$\square$ $\square$ $\boxed{0}$

8 9 1

No: of way 3 digit no are formed without repetation and last number as '0' = 8×9 = 72 ways

Case (ii)

Last Number = 2, 4, 6 or 8

$\square$ $\square$ $\square$

8 9 4

(2, 4, 6, 8)

No of ways 3 digit no are formed without repetation of last no as 2, 4, 6 or 8 $= 8 \times 9 \times 4$ $\frac{72}{\times 4}{288}$

$= 288$ ways

$\therefore$ Total no of 3 Digits $= 288 + 72$

$= 360$ ways

OP sum / and product

10. How many different bit strings of length seven are that so No of ways bit is chosen (0 or 1) = 2 ways

Sol No : of ways different bit string of length seven are choosen $\Big\} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^7$ ways

11. Prove that a set containing n elements has 2^n subsets?

Sol set has elements, a subset can be constructed in n diffs

i.e Take or do not take the 1^{st} elt
Take or do not take the 2^{nd} elt
- - - - - - - - - - - - -
- - - - - - - - - - - - -
take or do not take the n^{th} elt

$\Rightarrow$ Each step can be down in 2 different ways

Hence the possible no of subsets $= 2 \cdot 2 \cdot 2 \cdots 2 = 2^n$ (n times)

12) There are 6 multiple choice questions in an examination by the first 3 questions have 4 choices each and the next to 3 questions have 5 choices each, how many sequences of answers are possible?

4	4	4	5	5	5

Sol Total no : of ways answering all 6 question

11/2/23

Ex:- 1. Suppose there are 16 boys and 18 girls in a class How many ways we can select one student as a class representative ?

Sol

No. of boys = 16

No. of girls = 18

Total no. of ways a boy can be selected as a class representative = m = 16 ways

Total no. of ways a girl can be selected as a representative = n = 18 ways

∴ Total no. of ways either a boy or a girl can be choosen as a class representative is m+n ways

= 16+18

= 34 ways

2. In a town of Germany 8 Newspapers and 4 magazines are printed. Shyam want to subscribe 1 newspaper (or) 1 magazine. How many choices does he has ?

No. of newpapers = 8

No. of magazines = 4 } printed

Shyam can choose 1 news paper in m ways = 8

Shyam can choose 1 magazine in n ways = 4

∴ Total no of ways shyam can choose either News paper (or) a magazine = m+n ways

= 8+4

= 12 ways

3. 8 women and 6 men contest in a election

(i) In how many ways can people choose a leder ?

Sol No. of women Contestent's = 8

No. of men Contestents = 6

Total no of ways an women Contestent is Choosen as a leader = 8 ways

Total no of ways a men Contestent is Choosen as a leader = 6 ways

∴ Total no of ways can people select a leader (either men or women)

⇒ 8+6

⇒ 14 ways

(ii) In how many ways can people select '2' leaders a man and an women ?

Sol Total no of ways people choose women as a leader = 8 ways

Total no of ways people choose men as a leader = 6 ways

∴ Total no. of ways people can choose '2' leaders (a men and an women) = m×n ways

= 6×8

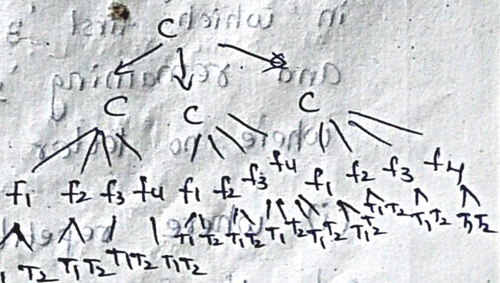
= 48 ways

4. Ramu visits an ice cream parlour to buy one He Sees that there are '3' Cones, '4' ice-creams flavours, and toppings How many choices of ice creams does Ramu has ?

No of Cones = 3

No of flovours = 4

NO of Toppings = 2



for each flavour he can choose any Toppings

∴ Total no of choices he can get

⇒ 3×4×2 = 24

5. How many three digit numbers are there which and have no repeated digits ?

for numbers to be even, they should end with $\}$ 0/2/4/6/8

Number of ways each of '3' digit is formed

(i) [with out repetitions and last digit as zero]

$\Rightarrow$ $\square$ $\square$ $\square$ $\}$ last no = 0

3rd digit 2nd digit 1st digit

$\Rightarrow$ $\underset{8}{\square}$ $\underset{9}{\square}$ $\underset{1}{\square}$

$\Rightarrow$ $8 \times 9 \times 1 = 72$ ways

(ii) No of way '3' digit no is formed [without repetitions and last digit is 2 (or) 4 (or) 6 (or) 8]

$\Rightarrow$ $\underset{8}{\square}$ $\underset{9}{\square}$ $\underset{4}{\square}$ $\}$ last digit = 2/4/6/8

$\Rightarrow$ $8 \times 9 \times 4 = 288$ ways

$\therefore$ Total no of '3' digit number formed / choosen

$\Rightarrow$ 288 + 72

$\Rightarrow$ 360 ways

6. How many to Construct sequence of 5 letters in which first '3' letters are English letter and remaining '2' are single digit number where no letter (or) digits can be repeated

(i) where repetitions are allowed

Sol No of letters to be Constructed = 5

No of Alphabits = 3 $\}$ 5

No of digits = 2

Case (i) with no repetitions $\underbrace{\_\ \_\ \_}_{\text{Alphabets }(A-Z)\ 26}\ \underbrace{\_\ \_}_{\text{single digits }(0-9)\ 10}$

$\underline{26}\quad \underline{25}\quad \underline{24}\quad \underline{10}\quad \underline{9}$

$\Rightarrow$ Total number of ways $= 26 \times 25 \times 24 \times 10 \times 9$

Case (ii) with repetitions

$\underline{26}\quad \underline{26}\quad \underline{26}\quad \underline{10}\quad \underline{10}$

$\Rightarrow$ Total no of ways $= 26 \times 26 \times 26 \times 10 \times 10$

10/1/23

pigeonhole principle :-

The principle states that "if there are more pigeons than the pigeonholes, then there must be at-least one pigeonhole with atleast two pigeons".

Definition :-

If 'K' is a positive integer and 'k+1' (or) more objects are placed in 'K' boxes, then there is atleast one box containing two (or) more of the objects

Generalization :-

If 'n' objects (pigeon's) are placed into $k\ (n>k)$ boxes (pigeonholes), then there is atleast one box containing atleast $[n/k]$ objects (or) $\left[\frac{n-1}{k}\right]+1$

Ex :- 1

Suppose there are 26 students and 5 cars to Transport then show that atleast one car must have 4 (or) more student

Sol $\left.\begin{array}{l} n = 26 \\ k = 5 \end{array}\right\}$ now applying pigeonhole principle

$$\Rightarrow \left[\frac{n-1}{k}\right]+1$$

$$\left[\frac{26-1}{5}\right]+1$$

$\Rightarrow \left[\frac{25}{5}\right]+1$

$\Rightarrow 5+1$

$\Rightarrow 6$

one Car Can Transport '6' Student

$\therefore$ one Car must have 4 (or) more passengers

Ex:-2 prove that if any '30' people are selected, then we may choose a subset of 'S' so that all '5' were born on same day of the week

Sol

$n = 30$

$k = 7$ (days of a week)

By applying pigeonhole principle

$\left[\frac{n-1}{k}\right]+1$

$\Rightarrow \left[\frac{30-1}{7}\right]+1$

$\Rightarrow \left[\frac{29}{7}\right]+1$

$\Rightarrow 4+1$

$\Rightarrow 5$

$\therefore$ 5 people were born on the same day of the week

Ex:-3 find the minimum number of teachers in a College to be sure that '4' of them are born in same month.

" If n pigeonholes are occupied by $k+1$ or more pigeons then atleast one pigeonhole is occupied by $k+1$ or more pigeons

from the given data $n = 12$ (months)

$k+1 = 4$

$k = 4-1 = 3$

$kn+1 = (3 \times 12)+1$

$\Rightarrow 36+1$

$\Rightarrow 37$

Ex:- 4 A box contains 10 blue balls, 20 red balls, 8 green balls, 15 yellow balls and 25 white balls. How many balls must we choose to ensure that we have 12 balls of same colour?

Sol

$n = 5$ (Colour ball given)

$k = 12$ (balls to be choosen)

$k+1 = 12$

$k = 12-1$

$k = 11$

$\Rightarrow kn+1 = (11 \times 5)+1$

$\Rightarrow 55+1$

$\Rightarrow 56$

$\therefore$ 56 balls has to be choosen to have 12 balls of same colour

Ex:- 5 Show that if 7 colours are used to paint 50 bicycles then atleast 8 of them will be of same colour?

Let colours denotes $k = 7$

Bicycles denotes $n = 50$

By applying pigeonhole principle $\left[\frac{n-1}{k}\right]+1$

$\Rightarrow \left[\frac{50-1}{7}\right]+1 \Rightarrow \left[\frac{49}{7}\right]+1 \Rightarrow 7+1$

$\Rightarrow 8$

$\therefore$ 8 bicycles will have same colour among 50 bicycles

Ex:- 6 Find the minimum number of socks that one needs to choose in order to get two pairs (4 socks) of same colour?

Sol '3' different Colours of socks

Assume $n = 3$, $k+1 = 4$

$\Rightarrow k = 4-1$

$\Rightarrow k = 3$

By applying the formula $kn+1$

$\Rightarrow 3 \times 3 + 1$

$\Rightarrow 9+1$

$\Rightarrow 10$

$\therefore$ 10 socks has to be Choosen in order to get '2' pairs of socks of same colour

** Ex:-7

Show If we pick 11 numbers we are sore that the sum of '2' picked numbers is 21

Given set = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \ldots 20\}$.

$\{1, 20\}$ $\{2, 19\}$ $\{3, 18\}$ $\{4, 17\}$ $\{5, 16\}$

$\{6, 15\}$ $\{7, 14\}$ $\{8, 13\}$ $\{9, 12\}$ $\{10, 11\}$

$$\underbrace{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 19}_{11 \text{ numbers}}$$

$2 + 19 = 21$

$20+1 = 21$

$\left(\frac{21-1}{20}\right)+1$

$n = 21$ (no of objects)

$k = 20$ (size)

$$\Rightarrow \left[\frac{n-1}{k}\right]+1$$

$$\Rightarrow \left[\frac{21-1}{20}\right]+1 \Rightarrow \left[\frac{20}{20}\right]+1$$

$\Rightarrow 1+1$

$\Rightarrow 2$

$\therefore$ sum of two picked numbers among '11' is '2'

Ex:-8 If we select any group of 1000 students on Campus then show that atleast '3' of them must have Same birthday.

Sol:- no. of pigeons n = No. of Students = 1000

no of pigeon holes k = No of days in an year = 365

By Generalization principle $\left[\frac{n-1}{k}\right]+1$

$\Rightarrow \left[\frac{1000-1}{365}\right]+1$

$\Rightarrow \left[\frac{999}{365}\right]+1$

$\Rightarrow [2.73]+1$

$\Rightarrow 3$

$\therefore$ atleast '3' students must have same birthday

17/1/23

Theorem :-

If k is a positive integer and 'n' pigeonholes are occupied by $(kn+1)$ pigeones, then prove that atleast one pigeonhole is occupied by $(k+1)$ or more pigeons

Sol Given that Number of pigeons $m = kn+1$

Number of pigeonholes $= n$

To prove atleast one pigeonhole is occupied by $(k+1)$ or more pigeons

Assume that, if atleast one pigeonhole is not occupied by $(k+1)$ or more pigeons $\rightarrow$ ①

$\Rightarrow$ Each pigeonholes contains atmost 'k' pigeons

so 'n' pigeonholes contains 'kn' pigeons, which is a contradiction to ①

Hence our assumption is wrong

Ex:-1 what is the minimum number of students required in a class to be sure that atleast '6' will receive the same grade, if there are '5' possible grades A, B, C, D, E

Sol

Number of pigeons m = Number of students

Number of pigenholes, n = Number of possible Grades = 5

By using principle of General pigeonhole we have $\left[\frac{m-1}{n}\right]+1 = 6$

$$\Rightarrow \left[\frac{m-1}{5}\right]+1 = 6$$

$$\Rightarrow m-1+5 = 30$$

$$\Rightarrow m+4 = 30$$

$$\Rightarrow m = 30-4$$

$$\Rightarrow m = 26$$

$\therefore$ minimum number of students required in a class = 26

Ex:-2 Find minimum number of integers to be selected from the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ so that sum of two integers is even

Number of pigeons, m = Number of integers ?

Number of pigeonholes, n = Number of subsets = 2

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\{1, 3, 5, 7, 9\}$ $\{2, 4, 6, 8\}$

Since atleast 2 numbers are to be selected for sum to be even

By General principle of pigeonhole $\left[\frac{m-1}{n}\right]+1 = 2$

$\Rightarrow m-1+2 = 4$ $\qquad \left[\frac{m-1}{2}\right]+1 = 2$

$\Rightarrow m+1 = 4$ $\qquad m-1+2 = 4$

$\Rightarrow m = 3$ $\qquad m+1 = 4$

$\therefore$ minimum number of integers to be selected from the set so that the sum of two integers is even $= \boxed{3}$

Permutations and Combinations :-

permutations :-

The different arrangements which can be made out of a given number of things, by taking some (or) all at a time is called " permutation "

Exent. ① Number of permutations of 'n' objects without repetitions

$$\boxed{p(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!}$$

② Number of permutations of n objects with repetitions

→ If it is required to find number of permtations that can be formed from a collection of 'n_1' objects of one kind, n_2 objects of other kind ...... n_k objects of k^{th} type with $n_1+n_2+n_3+ \cdots +n_k = n$ then we have

$$\boxed{\frac{n!}{n_1! * n_2! * n_3! * n_4! \cdots * n_k!}}$$

③ Number of permutations of 'r' objects among 'n' distinct objects

→ Suppose we are given 'n' objects (distinct) and wish to arrange 'r' objects then

$$\boxed{P(n,r) = \frac{n!}{(n-r)!}, \quad {}^nP_r = P(n,r)}$$

Note :-

1. A permutation is an act of arranging the objects
(or)
Numbers in an order (such that order of the objects matters)

2. A Combination is the way of selecting the objects
(or)
numbers from a group of objects (such that order of objects does not matter)

Ex :-
How many different strings (sequence) of length is Can be formed using the letter of the word FLOWER?

The given word FLOWER Contains 6 letters, all the letters are distinct (వేరువేరుగా ఉంటాయి)

Required number of strings formed are $P(6,4)$

$P(6,4)$, here $n=6$ $\quad P(n,r) = \frac{n!}{(n-r)!}$

r = length of Letters to be formed (4)

∴ The total number of strings of length '4' that Can be formed using the letter "FLOWER" = 360

$\Rightarrow \frac{6!}{(6-4)!} \Rightarrow \frac{6!}{2!} \Rightarrow \frac{6\times5\times4\times3\times\cancel{2}\times\cancel{1}}{\cancel{2}\times\cancel{1}}$

$\Rightarrow 30\times12 \Rightarrow 360$

2)720(360
6
12
12
0

Ex :- 2

How many number '2' Letter words which Can be formed by using the Letter of the word "GREAT"

The given word "GREAT" has '5' letters

All the Letters are distinct

$\therefore$ The required number of '2' letter words which Can be formed are

$P(n, r) \Rightarrow n = 5, \; r = 2$

$$P(5,2) = \frac{5!}{(5-2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} \Rightarrow \boxed{20}$$

Ex :- 3

Find number of words which Can be formed by using the Letter of the word "GREAT"

The given word "GREAT" Contains '5' letters

The required number of words which Can be formed are

$$P(n, n) = \frac{n!}{(n-n)!} = n!$$

here $n = 5$

$$= \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} \Rightarrow 5 \times 4 \times 3 \times 2$$

$$\Rightarrow \boxed{120}$$

Ex :- 4

Find the number of words which Can be formed by using the Letter of the word "SUCCESS"?

The given word "SUCCESS" has 7 Letters

out of which 'S' $\rightarrow$ 3

'C' $\rightarrow$ 2 and

'E' 'U' $\rightarrow$ 1 each

$\therefore$ The required number of permutations are

$$\frac{}{n_1! * n_2! * n_3! * \cdots * n_k!}$$

$$\Rightarrow \frac{7!}{3! * 2! * 1! * 1!}$$

$$\Rightarrow \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 * 2 * 1 * 1 * 1}$$

$$\Rightarrow \boxed{420}$$

Ex :- 5

How many of letter words can be formed by using the letter of the word "DIFFICULT"?

Sol The given word "DIFFICULT" has '9' letters out of which F, I → 2 remaining letters are one each

∴ The required number of permutations are

$$\frac{n!}{n_1! * n_2! * n_3! * \cdots * n_k!} \Rightarrow \frac{9!}{2! * 2! * 1! * 1! * 1! * 1! * 1!}$$

$$\frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 * 2 \times 1 * 1 * 1 * 1 * 1 * 1} \Rightarrow 9 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$\Rightarrow \frac{362880}{4}$$

$$\Rightarrow 90,720$$

Ex :- 6

It is required to seat '5' men and '4' women in a row, so that the women occupy even places. How many such arrangements are possible?

Sol

Number of men = 5

Number of women = 4

Given that '4' women can be seated in even places

⇒ '5' men can be seated in odd places

⇒ Correspondingly 5 men can be seated in odd places = 5! ways

'4' women can be seated in even places = 4! ways

m w m w m w m w m } men and women arranged in '9' positions

∴ Total number of permutations

$\Rightarrow 5! * 4!$

$\Rightarrow 120 * 24$

$\Rightarrow 2880$

18/1/23

Circular permutations :-

permutations in a circle are called as circular permutations.

→ Total number of ways of arranging the 'n' positions in a circle $= (n-1)!$

→ The number of circular permutations of 'n' different objects taken 'r' at a time $= \frac{{}^nP_r}{r!}$

→ If there is a situation of clockwise and anti-clock wise are identical then number of permutations $\Rightarrow \frac{n-1}{2}$

→ Number of circular permutations of 'n' things when 'p' things are alike a remaining are different is $\Rightarrow \frac{(n-1)!}{p!}$

Ex :-1

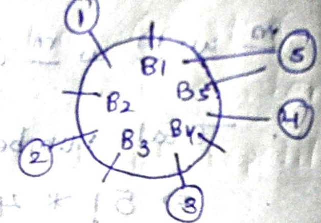
5 Boys and 5 girls are to be arranged around a circular table for a discussion, so that boys and girls sit alternatively. In how many ways they can be seated ?

Sol First arrange '5' boys around the table

→ Number of ways '5' boys can be arranged around a circular table $= (n-1)!$

$= (5-1)!$

$= 4!$ ways



→ Now boys are arranged already in a circular way.

In each of the arrangements there are '5' gaps, in which '5' girls are arranged (Linear permutation)

$\Rightarrow {}^5P_5 \Rightarrow 5!$ ways

∴ Total number of ways boys and girls can be arranged around a circular table is

$5! * 4! \Rightarrow 120 * 24$

$\Rightarrow 2880$ ways

Ex :- 2

A round table conference is to be held between '10' deligates of 10 countries. In how many ways can they be selected

(i) Two perticular deligates are always together

Sol: Number of deligates $= 10$

Since two perticular deligates are always together then treat them ~~two~~ as one unit and remaining people as '9' units

→ Number of ways '9' units can be arranged

$= (n-1)!$

$\Rightarrow (9-1)!$

$\Rightarrow 8!$ ways

→ Number of ways two deligates can be arranged $= {}^2P_2 \Rightarrow 2!$ ways

$\therefore$ Total number of arrangements $= 8! * 2!$ ways

$\Rightarrow 40320 * 2$

$\Rightarrow 80640$ ways

(ii) Two perticular deligates are either side of Chair person.

Given that '2' deligates are sitting either side of a Chair person

→ So treat these '3' as one unit and remaining '7' as other unit (Total 8 units)

→ Number of ways 8 units be arranged around a circullar table $= (8-1)! \Rightarrow 7!$ ways

→ Number of ways that '2' deligates can be arranged them selves $= {}^2P_2$

$= 2!$ ways

$\therefore$ Total number of ways (all the deligates can be arranged including a chair person is

$2! * 7!$ ways

$\Rightarrow 2 * 5040$

$\Rightarrow 10,080$ ways

Ex:-8

How many necklese of '12' beads each can be made from 18 beads of different colours?

Here clock-wise and anti-clockwise arrangements are same

$n = 18$ & $r = 12$

$$\frac{{}^nP_r}{2r} \Rightarrow \frac{{}^{18}P_{12}}{2(12)} \Rightarrow \frac{18!}{24} = \frac{18!}{6! * 24}$$

Ex :- 4

The password of a Computer system Consist of '8' distinct alphabetic Characters. Find number of passwords possible that (without Repetitions)

(i) End with string MATH

Sol Total number of alphabets = 26

→ Treat "MATH" as End of string

$\underbrace{\underline{19}\ \underline{20}\ \underline{21}\ \underline{22}}_{}$ $\underbrace{\underline{M}\ \underline{A}\ \underline{T}\ \underline{H}}_{}$

Left with remaining Alphabets without repetitions

Fixed '4' Alphabets at least '4' positions

∴ Total number of ways they can be arranged

$^nP_r = \frac{n!}{(n-r!)} \Rightarrow {}^{22}P_4 = \frac{22!}{(22-4)!}$

$\Rightarrow \frac{22!}{18!}$

$\Rightarrow$ 1,75,560 ways

(ii) No of ways a password can be formed begining with string "CREAM"?

Sol Total number of alphabets = 26

"CREAM" string is added at be begining of password

$\underbrace{\underline{C}\ \underline{R}\ \underline{E}\ \underline{A}\ \underline{M}}_{}$ $\underbrace{\underline{21}\ \underline{20}\ \underline{19}}_{}$

5 Alphabets are fixed at first '5' position

Remaining alphabets can be arranged without repetitions

Total number of ways password begin with String "CREAM" is

$^nP_r = \frac{n!}{(n-r)!} \Rightarrow \frac{21!}{(21-3)!} \Rightarrow \frac{21!}{18!}$

$\Rightarrow$ 7,980 ways

Ex:-5

In a playground '3' sisters and '8' other girls are playing together. In how many ways can all the girls be seated in a circular order so that '3' sisters are not seated together?

Given that there are '11'

3 sisters and '8' other girls

$\Rightarrow$ Total 11 girls

Number of ways to arrange these 11 girls in a circular manner $\Rightarrow (n-1)!$

$\Rightarrow (11-1)!$

$\Rightarrow 10!$ ways

$\rightarrow$ The '3' sisters can re arrange themselves in 3! ways

$\Rightarrow$ By Multiplication theorem 3 sisters come together along with 8 other girls (8! * 3!) ways

$\therefore$ Total number of ways in which arrangement can be made if '3' sisters are not seated together

$10! - (8! * 3!)$

$\Rightarrow 3628800 - (40320 * 6)$

$\Rightarrow 3628800 - 241920$

$\Rightarrow 3386880$ ways

Ex :- 6

Find number of ways in which '5' people A, B, C, D, E can be seated at a round table such that

(i) A and B always sit together

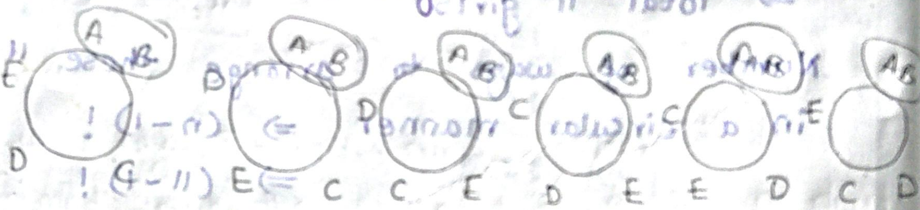
Sol If A and B always sit together we consider these two as one unit along with '3' other (C, D, E)

$\rightarrow$ So, we can arrange '4' people in a circle in $(n-1)!$ ways

$\Rightarrow (4-1)!$

$\Rightarrow 3!$

$\Rightarrow 6$ ways



$\rightarrow$ But in each of these arrangements A and B can arrange themselves in $2!$ ways

$\therefore$ Total number of ways in which '5' people can be arranged if A and B sit together

$\Rightarrow 6 * 2$

$\Rightarrow 12$ ways

19/1/22

Ex:-1 How many '3' digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits are repeated?

Sol $\square\square\square$

→ unit place is filled by 2, 4, 6 only

→ units place can be filled in 3 ways

→ Ten's place can be filled by any '6' digits in 6 different ways

→ Hundred place can be filled by any '6' digits in 6 different ways

∴ By multiplication rule, required number of '3' digit even numbers = $3 \times 6 \times 6$

$= 108$

Ex:-2

How many 5-digit telephone numbers can be formed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

Given that 5-digit telephone numbers always starts with 67

$\underbrace{\boxed{6}\,\boxed{7}}_{\text{Fixed}}\,\underbrace{\boxed{}\,\boxed{}\,\boxed{}}_{\text{'3' vacent places to be filled by digits 0-9}}$

→ units place can be filled any of digits from 0-9 except 6 and 7

→ Ten's place can be fixed by remaining 7 digits in 7 ways

→ Hundred's place can be filled by any of remaining '6' digits in 6 ways

∴ Required no. of ways in which 5 digit telephone no's can be constructed is

$8 \times 7 \times 6 = 336$ ways

Ex:- 3 If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$ then find 'x'

Sol

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

$$\Rightarrow \frac{1}{6!} + \frac{1}{7 \times 6!} = \frac{x}{8 \times 7 \times 6!}$$

$$\Rightarrow \frac{1}{6!}\left[1 + \frac{1}{7}\right] = \frac{x}{8 \times 7 \times 6!}$$

$$\Rightarrow \cancel{\frac{1}{6!}}\left[1 + \frac{1}{7}\right] = \frac{x}{8 \times 7} \times \cancel{\frac{1}{6!}}$$

$$\Rightarrow \frac{8}{7} = \frac{x}{8 \times 7} \Rightarrow 7x = 8 \times 8 \times 7$$

$$\Rightarrow x = \frac{8 \times 8 \times \cancel{7}}{\cancel{7}}$$

$$\Rightarrow \boxed{x = 64}$$

Ex:-4

How many 3-digit numbers have to be formed using the digits 1 to 9 if no digit is repeated?

→ 3-digit numbers has to be formed using 1-9 digits

→ Here order of the digits matters from 9 different digits take 3 at a time

$$\Rightarrow {}^{n}P_{r} = \frac{n!}{(n-r)!} \Rightarrow \frac{9!}{(9-3)!}$$

$$\Rightarrow \frac{9!}{6!}$$

$\Rightarrow \frac{9\times 8\times 7\times 6!}{6!}$

$\Rightarrow 504$

504 are required 3-digit numbers

Ex:- 5

If $^nP_4 = 20 * {^nP_2}$ then find n?

Sol

$^nP_4 = 20 * {^nP_2}$

$\therefore \frac{n!}{(n-4)!} = 20 * \frac{n!}{(n-2)!}$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 20 * \frac{n(n-1)(n-2)!}{(n-2)!}$

$\Rightarrow n(n-1)(n-2)(n-3) = 20 * n(n-1)$

$\Rightarrow \frac{\cancel{n}\cancel{(n-1)}(n-2)(n-3)}{\cancel{n}\cancel{(n-1)}} = 20$

$\Rightarrow (n-2)(n-3) = 20$

$\Rightarrow n^2 - 3n - 2n + 6 = 20$

$\Rightarrow n^2 - 5n + 6 = 20$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow n(n-7) + 2(n-7) = 0$

$\Rightarrow (n-7)(n+2) = 0$

$n = 7$ or $n = -2$ [permutations doesn't take negative value]

Since 'n' is a positive integer so $n = 7$

Ex:- 6

If $16 * {^nP_3} = 13 * {^{n+1}P_3}$ then find n?

Sol

$16 * {^nP_3} = 13 * {^{n+1}P_3}$

$\Rightarrow 16 * \frac{n!}{(n-3)!} = 13 * \frac{(n+1)!}{(n+1-3)!}$

$$\Rightarrow 16 * \frac{n!}{(n-3)!} = 13 * \frac{(n+1)!}{(n-2)!}$$

$$\Rightarrow \frac{16 * n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = \frac{13 * (n+1)n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$$

$$\Rightarrow 16 * \cancel{n}\cancel{(n-1)}(n-2) = 13 * (n+1)\cancel{n}\cancel{(n-1)}$$

$$\Rightarrow 16 * (n-2) = 13 * (n+1)$$

$$\Rightarrow 16n - 32 = 13n + 13$$

$$\Rightarrow 16n - 13n = 13 + 32$$

$$\Rightarrow 3n = 45$$

$$\Rightarrow n = \frac{45}{3}$$

$$\Rightarrow \boxed{n = 15}$$

Ex :- 7 If ${}^nP_4 : {}^nP_5 = 1:2$ then find 'n' ?

Sol

$${}^nP_4 : {}^nP_5 = 1:2$$

$$\therefore \frac{{}^nP_4}{{}^nP_5} = \frac{1}{2}$$

$$\Rightarrow \frac{\frac{n!}{(n-4)!}}{\frac{n!}{(n-5)!}} = \frac{1}{2}$$

$$\Rightarrow \frac{\frac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}}}{\frac{n(n-1)(n-2)(n-3)(n-4)\cancel{(n-5)!}}{\cancel{(n-5)!}}} = \frac{1}{2}$$

$$\Rightarrow \frac{\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}}{\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}(n-4)} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{n-4} = \frac{1}{2}$$

$$\Rightarrow n-4 = 2$$

$$\Rightarrow n = 2+4$$

$$\Rightarrow \boxed{n = 6}$$

Theorem :- prove that ${}^{n+1}P_{r+1} = (n+1) * {}^{n}P_r$

Proof :- To prove ${}^{n+1}P_{r+1} = (n+1) * {}^{n}P_r$

L.H.S :-

$${}^{n+1}P_{r+1} \Rightarrow \frac{(n+1)!}{(n+1-(r+1))!}$$

$$\Rightarrow \frac{(n+1)!}{(n-r)!}$$

$$\Rightarrow \frac{(n+1)(n+1-1)(n+1-2)(n+1-3)(n+1-4)\ldots * 3 * 2 * 1}{(n-r)!}$$

$$\Rightarrow (n+1) * \left[\frac{n * (n-1) * (n-2) * (n-3) * \ldots * 3 * 2 * 1}{(n-r)!}\right]$$

$$\Rightarrow (n+1) * \left[\frac{n!}{(n-r)!}\right]$$

$$\Rightarrow (n+1) * {}^{n}P_r$$

$$\Rightarrow R.H.S$$

$\therefore$ L.H.S = R.H.S

$${}^{n+1}P_{r+1} = (n+1) * {}^{n}P_r$$

Ex :- 8 Show that ${}^{10}P_3 = {}^{9}P_3 + 3 * {}^{9}P_2$

To prove ${}^{10}P_3 = {}^{9}P_3 + 3 * {}^{9}P_2$

L.H.S :-

$${}^{10}P_3 \Rightarrow \frac{10!}{(10-3)!} \Rightarrow \frac{10 \times 9 \times 8 \times 7!}{7!} \Rightarrow 720$$

R.H.S

$${}^{9}P_3 + 3 * {}^{9}P_2$$

$$\Rightarrow \frac{9!}{(9-3)!} + 3 * \left(\frac{9!}{(9-2)!}\right)$$

$$\Rightarrow \frac{9!}{6!} + 3 * \frac{9!}{7!}$$

$$\Rightarrow \frac{9 \times 8 \times 7 \times 6!}{6!} + 3 * \frac{9 \times 8 \times 7!}{7!}$$

$$\Rightarrow 9 \times 8 \times 7 + 3 * 9 \times 8$$

$$\Rightarrow 9 \times 8 [7 + 3]$$

$$\Rightarrow 9 \times 8 \times 10$$

$$\Rightarrow 720$$

$\therefore$ L.H.S = R.H.S

$$^{10}P_3 = {}^9P_3 + 3 * {}^9P_2$$

Ex:-9

How many words with or without dictionary meaning can be formed using all the letter of the word "JOULE"

Number of letter in the word JOULE = 5

→ Number of words which can be formed using all the '5' Letters (each letter exactly one time) is

$$^5P_5 \Rightarrow 5! \Rightarrow 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Ex:-10

'8' students are participating in a Competition. In how many ways can the first three prizes be won?

Number of Students = 8

Prizes to be won = 3

$\therefore$ no. of ways in which students gets the prizes

$$^8P_3 \Rightarrow \frac{8!}{(8-3)!} \Rightarrow \frac{8 \times 7 \times 6 \times 5!}{5!} \Rightarrow 336$$

Ex :- 11 If $^nP_r = 840$ $^nC_r = 35$ then find n ?

$^nP_r = 840$, $^nC_r = 35$

now $^nC_r = \frac{^nP_r}{r!} \Rightarrow 35 = \frac{840}{r!}$

$r! = \frac{840}{35} = 24$

$r! = 4 \times 3 \times 2 \times 1$

$\therefore \boxed{r = 4}$

Now

$^nP_r = 840$

$\Rightarrow {^nP_r} = 840 \Rightarrow {^nP_4} = 7 \times 6 \times 5 \times 4$

$\Rightarrow {^nP_4} = {^7P_4}$

$\boxed{n = 7}$

Theorem :-

prove that $\frac{^nC_r}{^{n-1}C_{r-1}} = \frac{n}{r}$ when $1 \le r \le n$

proof :- To prove that $\frac{^nC_r}{^{n-1}C_{r-1}} = \frac{n}{r}$

L.H.S $= \frac{^nC_r}{^{n-1}C_{r-1}} \Rightarrow \frac{\frac{n!}{(n-r)! * r!}}{\frac{(n-1)!}{(n-1-r+1)! * (r-1)!}}$

$\Rightarrow \frac{n!}{(\cancel{n-r})! * r!} \times \frac{(\cancel{n-r})! * (r-1)!}{(n-1)!}$

$\Rightarrow \frac{n(\cancel{n-1})(\cancel{n-2})(\cancel{n-3})\cdots * (\cancel{r-1}) * (\cancel{r-2}) * (\cancel{r-3})\cdots}{r(\cancel{r-1})(\cancel{r-2})(\cancel{r-3})\cdots * (\cancel{n-1})(\cancel{n-2})(\cancel{n-3})\cdots}$

$\Rightarrow \frac{n}{r} = R.H.S$

L.H.S = R.H.S

$\frac{^nC_r}{^{n-1}C_{r-1}} = \frac{n}{r}$ when $1 \le r \le n$

Ex :- 12

If ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 3:4:5$ then find 'r' ?

Sol $\quad {}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 3:4:5$

$$\therefore \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{4} \Rightarrow \frac{n!}{(n-r+1)!(r-1)!} * \frac{(n-r)!\,r!}{n!} = \frac{3}{4}$$

$$\Rightarrow \frac{\cancel{(n-r)}\cancel{(n-r-1)}\cancel{(n-r-2)}\cancel{(n-r-3)}\cdots * r\cancel{(r-1)}\cancel{(r-2)}\cancel{(r-3)}\cdots}{(n-r+1)\cancel{(n-r)}\cancel{(n-r-1)}\cdots * \cancel{(r-1)}\cancel{(r-2)}\cancel{(r-3)}\cdots} = \frac{3}{4}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{3}{4}$$

$\Rightarrow 4r = 3n - 3r + 3$

$\Rightarrow 4r + 3r = 3n + 3$

$\Rightarrow 7r = 3n + 3$

$\Rightarrow 7r - 3n = 3 \longrightarrow (1)$

Now $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{4}{5}$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} * \frac{(n-r-1)!\,(r+1)!}{n!} = \frac{4}{5}$$

$$\Rightarrow \frac{(n-r-1)(n-r-2)\cdots * (r+1)(r)(r-1)(r-2)\cdots}{(n-r)(n-r-1)(n-r-2)\cdots * r(r-1)(r-2)\cdots} = \frac{4}{5}$$

$$\Rightarrow \frac{r+1}{n-r} = \frac{4}{5}$$

$\Rightarrow 5r + 5 = 4n - 4r$

$\Rightarrow 9r - 4n = -5 \longrightarrow (2)$

Now multiply equation (1) with '4' and (2) with '3' we get

$4 * (7r - 3n) = 4*3$ and $3 * (9r - 4n) = 3 * -5$

$28r - 12n = 12 \rightarrow (3)$ and $27r - 12n = -15 \rightarrow (4)$

Subtracting equation ④ from ③ we get

$$28r - 12n = 12$$
$$27r - 12n = -15$$

$$r = 27$$

$$\boxed{r = 27}$$

1) Prove that $1 \le r \le n$, ${}^nP_r = n \cdot {}^{(n-1)}P_{(r-1)}$

We know that ${}^nP_r = \frac{n!}{(n-r)!}$

Consider R.H.S :-

$= n \cdot {}^{(n-1)}P_{(r-1)}$

$= n \left[\frac{(n-1)!}{[(n-1)-(r-1)]!}\right] \Rightarrow n\left[\frac{(n-1)!}{(n-1-r+1)!}\right]$

$= \frac{n(n-1)!}{(n-r)!}$

$= \frac{n!}{(n-r)!}$

$= {}^nP_r$

$\therefore$ L.H.S = R.H.S

Hence when $1 \le r \le n$ ${}^nP_r = n\,{}^{(n-1)}P_{(r-1)}$

2) Prove that $1 \le r \le n$ ${}^nP_r = {}^{(n-1)}P_r + r \cdot {}^{(n-1)}P_{(r-1)}$

$= \frac{(n-1)!}{(n-1-r)!} + r\left[\frac{(n-1)!}{(n-1)-(r-1)!}\right]$

$= \frac{(n-1)!}{(n-1-r)!} + \frac{r \cdot (n-1)!}{(n-1)-(r-1)!}$

$n(n-1)! = n!$

$= \frac{(n-1)!}{(n-1-r)!} + \frac{r(n-1)!}{(n-1+1-r)!}$

$$= \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-r)!\,(n-1)!}{(n-1-r)!\,(n-r)!} + r\frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-r)(n-1) + r(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!\,[n-\cancel{r}+\cancel{r}]}{(n-r)!}$$

$$= \frac{(n-1)!\,(n)}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^nP_r$$

Hence L.H.S = R.H.S

Hence for $1 \le r \le n$, ${}^nP_r = {}^{n-1}P_r + r\,{}^{(n-1)}P_{(r-1)}$

Combination Theorems :-

Theorem :- 1

If n, r positive integers with $0 \le r \le n$ then ${}^nC_r = {}^nC_{(n-r)}$

Proof :-

To prove ${}^nC_r = {}^nC_{(n-r)}$

Consider R.H.S :- ${}^nC_{(n-r)}$

W.K.T $\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$

$$\Rightarrow \frac{n!}{(n-(n-r))! \times (n-r)!}$$

$$\Rightarrow \frac{n!}{(n-n+r)! \times (n-r)!}$$

$$\Rightarrow \frac{n!}{r! \times (n-r)!}$$

$$\Rightarrow {}^{n}C_{r}$$

L.H.S

$\therefore$ L.H.S = RHS

Hence for n, r be positive integers ${}^{n}C_{r} = {}^{n}C_{(n-r)}$

Binomial Coefficient theorem :-

Theorem :-

Prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Proof :- To prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Consider L.H.S $= {}^{n}C_{r} + {}^{n}C_{r-1}$

$$\Rightarrow \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)! \times (n-(r-1))!}$$

$$\Rightarrow \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)! \times (n-r+1)!}$$

$$\Rightarrow \frac{n! \times (n-r+1)}{r! \times (n-r)! \times (n-r+1)} + \frac{n! \times r}{(r-1)! \times (n-r+1) \times r}$$

$$\boxed{\begin{aligned}&(n-r)! \times (n-r+1) = (n-r+1)! \\ &r \times (r-1)! = r!\end{aligned}}$$

$$\Rightarrow \frac{n! * (n-r+1)}{r! * (n-r+1)!} + \frac{n! * r}{r!\,(n-r+1)!}$$

$$\Rightarrow \frac{n!}{r!\,(n-r+1)} \left[n-\cancel{r}+1+\cancel{r}\right]$$

$$\Rightarrow \frac{n!\,(n+1)}{r! \times (n-r+1)!}$$

$$\Rightarrow {}^{n+1}C_r \qquad \cancel{\frac{(n+1)n!}{(n+1-r)!\,r!}}$$

R.H.S

$\therefore$ L.H.S = R.H.S

Hence ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

Ex:- 1 In how many ways 5 Sportsmen can be selected from a group of 10 ?

sol

5 sports men can be selected from a group of '10' in ${}^{10}C_5$ ways

$\therefore$ Total number of ways of selecting '5' sportsmen is

$${}^{10}C_5 = \frac{10!}{5!\,(10-5)!}$$

$${}^{n}C_r = \frac{n!}{r!\,(n-r)!}$$

$n = 10$

$r = 5$

$$= \frac{10!}{5!\,(5)!}$$

$$= \frac{\cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times 6 \times \cancel{5!}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times 2 \times 1 \times 5!}$$

$$= \frac{2 \times 3 \times \cancel{2} \times 7 \times 6}{\cancel{2} \times 1}$$

$$= 2 \times 3 \times 6 \times 7$$

$= 252$ ways

Ex:-2) out of '7' Consonents and '4' vowels, how many words of 3 Consonents and '2' vowels Can be formed?

Sol '3' Consonents out of '7' Can be selected in $^{7}C_{3}$ ways

'2' vowels out of '~~5p~~' Can be selected in $^{4}C_{2}$ ways

∴ Total no. of ways words Can be formed

$$= {}^{7}C_{3} \times {}^{4}C_{2}$$

$$= \frac{7!}{3! \times (7-3)!} \times \frac{4!}{2!(4-2)!}$$

$$= \frac{7 \times \cancel{6}^{2} \times 5 \times \cancel{4!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{(4)!}} \times \frac{\cancel{4}^{2} \times 3 \times \cancel{2!}}{\cancel{2} \times 1 \times \cancel{(2)!}}$$

$$= 7 \times 5 \times 2 \times 3$$

$$= 35 \times 6$$

$$= 210$$

∴ Total number of words Can be formed = 210

Ex:-3) How many (i) Lines (ii) Triangles Can be drawn through '8' points on a circle?

Sol

(i) Two points are needed to draw a line

∴ number of lines that Can be drawn from '8' points $= {}^{8}C_{2}$

$$= \frac{8!}{2!(8-2)!}$$

$$= \frac{\cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{2} \times 1 \times \cancel{6!}}$$

$= 4 \times 7$

$= 28$ lines

(iii) '3' points are needed to draw a triangle

$\therefore$ number of triangles that can be drawn from '8' points is

$^{8}C_{3} = \dfrac{8!}{3! \times (8-3)!}$

$= \dfrac{\cancel{8} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{3 \times 2} \times 1 \times \cancel{5!}}$

$= 4 \times 7 \times 2$

$= 28 \times 2$

$= 56$ triangles

Ex 4:- A polygon has 44 diagonals. Find the number of sides?

Sol Number of polygons of 'n' sides $= \boxed{^{n}C_{2} - n}$

$\Rightarrow \boxed{\dfrac{n(n-3)}{2}}$

Here from given data diagonals $= 44$

$44 = \dfrac{n(n-3)}{2}$

$\Rightarrow 44 \times 2 = n^2 - 3n$

$\Rightarrow 88 = n^2 - 3n$

$\Rightarrow n^2 - 3n - 88 = 0$

$\Rightarrow n^2 - 11n + 8n - 88 = 0$

$\Rightarrow n(n-11) + 8(n-11) = 0$

$\Rightarrow (n-11)(n+8) = 0$

$n-11=0 \quad n+8=0$

$n=11$ (or) $n=-8 \rightarrow$ not valid

$\therefore n=11$

If a Polygon has 44 diagonals then number of sides $=11$

Ex-5. In how many ways a student can choose '5' Courses out of '9' Courses if '2' Courses are compulsory for every student?

Sol Total number of Courses $=9$

out of '9' courses '2' Courses are Compulsory

So Courses left for student $=7$ $(9-2)$

Hence total number of ways by which '3' (remaining out of 5 Courses) Can be selected from $= {}^{7}C_{3}$

$$= \frac{7!}{3! * (7-3)!} \Rightarrow = \frac{7 \times 6 \times 5 \times \cancel{4!}}{3 \times 2 \times 1 \times \cancel{4!}}$$

$$\frac{7 \times \cancel{6}^{2} \times 5}{\cancel{3} \times \cancel{2} \times 1} = 35$$

Ex :- 6

Evaluate (i) ${}^{10}C_{3}$ (ii) ${}^{60}C_{60}$

we know that, ${}^{n}C_{r} = \frac{n!}{(n-r)! * r!}$

$$\Rightarrow {}^{10}C_{3} = \frac{10!}{3! * (10-3)!}$$

$$\Rightarrow \frac{10!}{3! * 7!}$$

$$\Rightarrow \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4} \times \cancel{7!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{7!}}$$

$\Rightarrow 10 \times 3 \times 4$

$\Rightarrow 30 \times 4$

$\Rightarrow 120$

(ii) $^{60}C_{60}$

W.K.T, $^nC_r = \frac{n!}{(n-r)! * r!}$

$\Rightarrow {}^{60}C_{60} = \frac{60!}{60! * (60-60)!}$

$\Rightarrow \frac{60!}{60! \times 0!} = \frac{\cancel{60!}}{\cancel{60!} \times 1} = 1$

EX:-7 In how many ways '5' letters can be posted in '3' post boxes if any number of letters can be posted in all the '3' post boxes ?

Given that, number of letters to be posted = 5

number of post boxes = 3

Since each letter can be posted to any of the '3' post boxes, it can be done in '3' ways

∴ There are 5 letters so total number of ways they can be posted

$= 3 \times 3 \times 3 \times 3 \times 3$

$= 243$ ways

Ex :- 8 Find 'n' if ${}^nC_4 = 210$

Sol To find the value of n when ${}^nC_4 = 210$

we know that $\boxed{{}^nC_r = \dfrac{n!}{r!(n-r)!}}$

$$\Rightarrow {}^nC_4 = \frac{n!}{4! * (n-4)!}$$

$$\Rightarrow 210 = \frac{n!}{4! * (n-4)!}$$

$$\Rightarrow 210 = \frac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{4*3*2*1 * \cancel{(n-4)!}}$$

$$\Rightarrow 210 = \frac{n(n-1)(n-2)(n-3)}{4*3*2*1}$$

$$\Rightarrow 210 = \frac{n(n-1)(n-2)(n-3)}{24}$$

$$\Rightarrow \underbrace{n(n-1)(n-2)(n-3)}_{\text{Product of '4' Consecitive Integers}} = 210 * 24$$

$\Rightarrow 210 * 24$

10 21 8 3

7 3 9

$$\Rightarrow 10(10-1) * (10-2)(10-3) = 210 * 24$$

$\Rightarrow 10 * 9 * 8 * 7$ which is equivalent to $210 * 24$

$$\Rightarrow \boxed{n = 10}$$

Ex :- 9

At a party, each person shake hands with every other person If there are totally '66' handshakes how many people are there at the party ?

Sol :- Let 'n' be number of persons from given data

$^nC_2 = 66$

$\Rightarrow \frac{n!}{2!(n-2)!} = 66 \Rightarrow \frac{n(n-1)\cancel{(n-2)!}}{2*1*\cancel{(n-2)!}} = 66$

$\Rightarrow \frac{n(n-1)}{2} = 66 \Rightarrow n^2 - n = 66 * 2$

$\Rightarrow n^2 - n - 132 = 0$ $\boxed{12 \times 11 = 132}$

$\Rightarrow n^2 - 12n + 11n - 132 = 0$

$\Rightarrow n(n-12) + 11(n-12) = 0$

$(n+12) = 0$ (or) $(n+11) = 0$

$\boxed{n = 12}$ $\boxed{n = -11} \rightarrow$ not valid solution

$\therefore n = 12$

Hence number of people in party = 12

Ex :- 10

Alice has '6' chocolates, all of them are of different flavours she wants to give '2' of her chocolates to her friends. How many different combination of chocolates can Alice make from '6' chocolates ?

Sol Number of chocolates Alice has = 6

Number of chocolates she wants to give to her friends = 2

By using Binomial Coefficient formula

$^nK = \frac{n!}{K!(n-K)!}$ where $n \geq K \geq 0$

here $n = 6$ & $K = 2$

$$\Rightarrow \frac{6!}{2! * (6-2)!}$$

$$\Rightarrow \frac{\cancel{6}^{3} \times 5 \times \cancel{4!}}{\cancel{2} * 1 * \cancel{4!}}$$

$\Rightarrow 3 \times 5$

$\Rightarrow 15$

$\therefore$ Alice Can make '15' different Combinations of chocolates.

Ex :- 11

In how many ways Can we distribute '12' identical pencils to '5' Children so that every child gets atleast one pencil ?

Sol Given that Number of Pencils = 12

Number of children = 5

and Every child gets at least one pencil ≥ 1

$\Rightarrow$ First we distribute one pencil to each child

$\Rightarrow$ the remaning pencils $(12 - 5 = 7)$ Can be distributed to 5 Children

$$\Rightarrow \boxed{C(r+n-1, r)}$$

here $r = 7$ $n = 5$

$\Rightarrow C(7+5-1, 7)$

$\Rightarrow C(7+4, 7)$

$\Rightarrow C(11, 7) \Rightarrow \dfrac{11!}{7! * (11-7)!}$

$$\Rightarrow \frac{11!}{7! * 4!}$$

$$\Rightarrow \frac{11 \times 10 \times \cancel{9}^{3} \times \cancel{8}^{2} \times \cancel{7!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{7!}}$$

$\Rightarrow 11 \times 10 \times 3$

$\Rightarrow 330$

$\therefore$ number of ways to distribute pencils to '5' children $= 330$

EX :- 12

If $p(n-r) = 2520$ and $c(n, r) = 21$ then what is the value of $c(n+1, r+1)$?

Sol

Given that $P(n, r) = 2520$, $c(n, r) = 21$

$$P(n, r) = \frac{n!}{(n-r)!} = 2520 \rightarrow ①$$

$$C(n, r) = \frac{n!}{r! * (n-r)!} = 21 \rightarrow ②$$

Comparing ① & ② equations, Substitue ① in ②

$$\Rightarrow \frac{2520}{r!} = 21$$

$$\Rightarrow r! = \frac{\cancel{2520}^{120}}{\cancel{21}}$$

$\Rightarrow r! = 120$

$\therefore \boxed{r = 5}$

$\Rightarrow P(n, r) = 2520$

$\Rightarrow P(n, 5) = 2520$

$$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-5)} = 2520$$

$$\Rightarrow \frac{n*(n-1)(n-2)(n-3)(n-4)\cancel{(n-5)!}}{\cancel{(n-5)!}} = 2520$$

$$\Rightarrow n*(n-1)(n-2)(n-3)(n-4) = \underbrace{7*6*5*4*3}_{2520}$$

$\therefore \boxed{n=7}$

we know that $n=7$, $r=5$

$$c(n+1, r+1) = c(7+1, 5+1)$$
$$= c(8, 6)$$

$$\Rightarrow \frac{8!}{6!(8-6)!} \Rightarrow \frac{8\times 7\times \cancel{6!}}{2!*\cancel{6!}} \Rightarrow \frac{8\times 7}{2\times 1} \Rightarrow 28$$

$\therefore c(n+1, r+1) = 28$

Ex :- 13

Solve for 'n' if $^nP_5 = 42 * {}^nP_3$ where $n>4$

Sol given condition ^nP_r $n>r$

$$^nP_5 = 42 * {}^nP_3$$

[we know that $^nP_r = \frac{n!}{(n-r)!}$]

$$\Rightarrow \frac{n!}{(n-5)!} = 42 * \frac{n!}{(n-3)!}$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)(n-4)\cancel{(n-5)!}}{\cancel{(n-5)!}} = 42 * \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$$

$$\Rightarrow \cancel{n}\cancel{(n-1)}\cancel{(n-2)}(n-3)(n-4) = 42 * \cancel{n}\cancel{(n-1)}\cancel{(n-2)}$$

$$\Rightarrow (n-3)(n-4) = 42$$

$$\Rightarrow n^2-3n-4n+12 = 42$$

$$\Rightarrow n^2-7n+12-42=0$$

$$\Rightarrow n^2-7n-30=0$$

$$\Rightarrow n^2-10n+3n-30=0$$

$$\Rightarrow n(n-10)+3(n-10)=0$$

$$\Rightarrow (n+3)=0 \text{ or } (n-10)=0$$

$\therefore n = 10$ for $^nP_5 = 42 \times {^nP_3}$

Ex :- 14

Solve for n if $\dfrac{^nP_4}{^{n-1}P_4} = \dfrac{5}{3}$, $n > 4$

Sol

$$\boxed{^nP_r = \frac{n!}{(n-r)!}}$$

$$\Rightarrow \frac{\dfrac{n!}{(n-4)!}}{\dfrac{(n-1)!}{(n-1-4)!}} = 5/3$$

$$\Rightarrow \frac{\dfrac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}}}{\dfrac{(n-1)(n-2)(n-3)(n-4)\cancel{(n-5)!}}{\cancel{(n-5)!}}} = \frac{5}{3}$$

$$\Rightarrow \frac{n}{(n-4)} = \frac{5}{3}$$

$\Rightarrow 3n = 5(n-4)$

$\Rightarrow 3n = 5n - 4(5)$

$\Rightarrow 5n - 3n - 20 = 0$

$\Rightarrow 2n - 20 = 0$

$\Rightarrow 2n = 20$

$\Rightarrow n = \dfrac{20}{2}$

$\Rightarrow \boxed{n = 10}$

So, $n = 10$ for $\dfrac{^nP_4}{^{n-1}P_4} = \dfrac{5}{3}$ when $n > 4$

Ex :- 15

Solve algebraically $\frac{(n-1)!}{(n-3)!} = 30$ to find value of 'n' when $n \geq 3$

Sol To solve $\frac{(n-1)!}{(n-3)!} = 30$

$$\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 30$$

$\Rightarrow (n-1)(n-2) = 30$

$\Rightarrow n^2 - 2n - n + 2 = 30$

$\Rightarrow n^2 - 3n + 2 = 30$

$\Rightarrow n^2 - 3n + 2 - 30 = 0$

$\Rightarrow n^2 - 3n - 28 = 0$

$\Rightarrow n^2 - 7n + 4n - 28 = 0$

$\Rightarrow n(n-7) + 4(n-7) = 0$

$n + 4 = 0$ or $n - 7 = 0$

$n = -4$ or $n = 7$

$\downarrow$

not valid solution

$\therefore n = 7$ for $\frac{(n-1)!}{(n-3)!} = 30$ when $n \geq 3$

Ex :- 16

Simplify $\frac{n(n+1)!}{(n-1)!}$

Sol To Simplify $\frac{n(n+1)!}{(n-1)!}$

$$\Rightarrow \frac{n(n+1)\, n\, \cancel{(n-1)!}}{\cancel{(n-1)!}}$$

$\Rightarrow n(n+1) * n$

$\Rightarrow (n^2+n)\, n$

$\Rightarrow n^3 + n^2$

Ex :- 17 If $P(11, 4) = 7920$ then find $P(11, 7)$.

Sol: Given that $P(11, 4) = 7920$
we have to find $P(11, 7)$.

$$\boxed{P(n, n-r) = \frac{(n-r)!}{r!} * P(n, r)}$$

$$\Rightarrow P(11, 7) = P(11, 11-7)$$

$$\cancel{P(11,4)} \cancel{=} \cancel{\frac{11!}{(11-4)!}} = \frac{(11-4)!}{4!} * P(11, 4)$$

$$\Rightarrow \frac{7!}{4!} * 7920 \quad (\text{given that } P(11, 4) = 7920)$$

$$\Rightarrow \frac{7 * 6 * 5 * \cancel{4!}}{\cancel{4!}} * 7920$$

$$\Rightarrow 210 * 7920$$

$$\Rightarrow 166320$$

$$\therefore P(11, 7) = 166320$$

Ex :- 18

Solve for 'n' $n \in \omega$ $3P(n, 4) = P(n-1, 5)$

Sol: Given that $n \in \omega$ and $3P(n, 4) = P(n-1, 5)$.

$$\Rightarrow 3 * \frac{n!}{(n-4)!} = \frac{(n-1)!}{(n-1-5)!}$$

$$\Rightarrow \frac{3 * n\cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}\cancel{(n-4)!}}{\cancel{(n-4)!}} = \frac{\cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}(n-4)(n-5)\cancel{(n-6)}}{\cancel{(n-6)!}}$$

$$\Rightarrow 3 * n = (n-4)(n-5)$$

$$\Rightarrow 3n = n^2 - 4n - 5n + 20$$

$$\Rightarrow 3n = n^2 - 9n + 20$$

$$\cancel{\Rightarrow} \quad \cancel{3n} =$$

$\Rightarrow n^2 - 3n - 9n + 20 = 0$

$\Rightarrow n^2 - 12n + 20 = 0$

$\Rightarrow n^2 - 10n - 2n + 20 = 0$

$\Rightarrow n(n-10) - 2(n-10) = 0$

$\Rightarrow (n-2) = 0$ (or) $(n-10) = 0$

$n = 2$ (or) $n = 10$

Not a valid answer

Since n is < r which is not Correct as per permutation / Combination rule, so we neglect $n=2$

Therefore $n = 10$ is a valid solution for new

$3P(n,4) = P(n-1, 5)$

Ex:49 Solve for 'n' if $3C(n,2) = P(6,2)$

Sol To solve 'n' when $3C(n,2) = P(6,2)$

$\Rightarrow 3 * \frac{n!}{2!(n-2)!} = \frac{6!}{(6-2)!}$

$\Rightarrow 3 * \frac{n(n-1)(n-2)!}{2 * 1 * (n-2)!} = \frac{6!}{4!}$

$\Rightarrow 3 * \frac{n(n-1)}{2} = \frac{6 * 5 * 4!}{4!}$

$\Rightarrow \cancel{3} * \frac{n(n-1)}{2} = \cancel{30}\ 10$

$\Rightarrow n(n-1) = 2 * 10$

$\Rightarrow n^2 - n = 20$

$\Rightarrow n^2 - n - 20 = 0$

$n^2 - 5n + 4n - 20 = 0$

$n(n-5) + 4(n-5) = 0$

$(n+4) = 0$ (or) $n - 5 = 0$

$\boxed{n = -4}$ $\boxed{n = 5}$

Not valid solution is a valid solution

Therefore $n=5$ for $3C(n,2) = P(6,2)$

Ex :- 20

Solve algebrically $\frac{n!}{(n-2)! * 4!} = 10$

Given that $\frac{n!}{(n-2)! * 4!} = 10$

$$\Rightarrow \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\, 4*3*2*1} = 10$$

$$\Rightarrow \frac{n(n-1)}{24} = 10$$

$$\Rightarrow n(n-1) = 10 * 24$$

$$\Rightarrow n^2 - n = 240$$

$$\Rightarrow n^2 - n - 240 = 0$$

$$\Rightarrow n^2 - 16n + 15n - 240 = 0$$

$$\Rightarrow n(n-16) + 15(n-16) = 0$$

$$\Rightarrow (n+15) = 0 \text{ (or) } (n-16) = 0$$

$n = -15$ (or) $n = 16$

Not valid answer

So, $n=16$ is valid solution for $\frac{n!}{(n-2)! * 4!} = 10$

$10 \times 24 = 40 + 200 = 240$

Ex :- 21

How many Committes of '5' with a chair person Can be selected from '12' persons ?

Sol Total number of persons = 12

Committe Consists of '5' members

→ A chairperson Can be selected among 12 persons in 12 ways

→ So remaining '4' members Committe Can be selected in $^{11}C_4$ ways.

∴ Total number of Committes of '5' with 12 persons Can be formed = $12 * {}^{11}C_4$

$$= 12 * \frac{11!}{4! * (11-4)!}$$

$$\Rightarrow 12 * \frac{11!}{4! * 7!}$$

$$\Rightarrow 12 * \frac{11 * 10 * \cancel{9}^3 * \cancel{8}^2 * \cancel{7!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 * \cancel{7!}}$$

$\Rightarrow 12 * 11 * 10 * 3$

$\Rightarrow 110 * 36$

$\Rightarrow 3,960$

Ex :- 22

In how many ways we can arrange the letters of the word "TORONTO" such that

(i) The word begins with 'T' ?

(ii) The word begins with 'O' ?

(iii) the word ends with 'N' ?

Sol Given word is "TORONTO" which has '7' Letters

(i) the number of ways we can arrange the word that starts with 'T' is T _ _ _ _ _ _

= Doesn't Count

$\boxed{T}$ $\underbrace{\text{ORONTO}}_{\text{Remaning 6 Letters}}$

In Remaining 6 Letters.

O's — 3 (3 times)

R — 1 (1 time)

N — 1 (1 time)

$$\Rightarrow \frac{6!}{3! * 1! * 1!} \Rightarrow \frac{6*5*4*\cancel{3}*2*1}{\cancel{3!}} \Rightarrow 30 \times 4 \Rightarrow 120$$

(ii) Number of ways we can arrange the word that starts with 'O' is O _ _ _ _ _ _

$$O\underbrace{TRONTO}_{\text{Reamaning 6 Letters}}$$

T - 2 (2 times)
R - 1 (1 time)
N - 1 (1 time)

$$\Rightarrow \frac{6!}{2! * 1! * 1!} \Rightarrow \frac{6*5*4*3*\cancel{2}*1}{\cancel{2}*1} \Rightarrow 360$$

(iii) Number of ways we can arranage the word that ~~star~~ Ends with N is _ _ _ _ _ _ N

$$\underbrace{TOROTO}_{\text{Reamaning 6 letters}}N$$

T - 2 (2 times)
O - 3 (3 times)
R - 1 (1 time)

$$\Rightarrow \frac{6!}{2!\,3!\,1!} \Rightarrow \frac{\overset{2}{\cancel{6}}\times 5\times 4\times \cancel{3}\times \cancel{2}\times 1}{\cancel{2}\times 1\times \cancel{3}\times \cancel{2}\times 1} = 60$$

	Permutations	Combinations
1.	A permutation is an arrangement of items in a particular order. ~~Notice~~ which order matters	A Combination is an Selecting / arrangement of items in ~~a partice~~ which order does not matter
2.	Represents arrangement	Represents grouping / selection
3.	many permutations can be derived from a single Combination	only one Combination can be derived with one permutation
4.	Denoted by ^nP_r	Denoted by ^nC_r
5.	multiple permutation from a single Combination	single Combination from a single permutation
6.	The number of ~~Combinations~~ permutations of n things chosen r at a time is found using $^nP_r = \frac{n!}{(n-r)!}$	The number of Combination of n things chosen r at a time is found using $^nC_r = \frac{n!}{r!(n-r)!}$
7.	Clue words are arrangement, schedule, order	Clue words are group, sample, select
8.	The number of ways to arrange things	The number of ways to choose things
9.	Sequence does matter 123 is not the same as 321	order does not matter 123 is the same as 321