# MASTER OF COMPUTER APPLICATIONS DEGREE EXAMINATION, JUNE - 2022

## FIRST SEMESTER

## Paper - MCA 101 : DISCRETE MATHEMATICAL STRUCTURES

*(Under C.B.C.S Revised Regulations w.e.f. 2020-2021)*

*(Common Paper to University and all Affiliated Colleges)*

**Time : 3 Hours** **Max. Marks : 70**

## PART - A

## (Compulsory)

Answer any **FIVE** of the following Questions. Each Question carries **2** marks. **(5×2=10)**

**1.** a) Show that $\neg(p \vee q)$ *and* $\neg p \wedge \neg q$ are logically equivalent.

b) Let Q(x) be the statement "x < 2." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers?

c) Give a recursive definition of $a^n$, where a is a nonzero real number and n is a nonnegative integer.

d) Write about Well-Formed formulae in Propositional Logic.

e) How many permutations of the letters ABCDEFGH contain the string ABC?

f) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

g) What is the solution of the recurrence relation $a_n = 6a_{n-1}$-l-$9a_{n-2}$ With $a_0 = 1$ and $a_1 = 6$?

h) Define linear nonhomogeneous recurrence relation of degree k with constant coefficients.

i) Write about Dirac's Theorem with proof.

j) Give the difference between Euler path and circuit.

## PART - B

Answer any **ONE** full question from each Unit. Each Question carries **12 marks.**

**(5×12=60)**

### UNIT - I

**2.** i) Obtain principal conjunctive normal form (PCNF) for the formula $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

ii) Show that the following is inconsistent $P \rightarrow Q, R \rightarrow S, P \vee R, \sim(Q \vee S)$.

**(OR)**

**3.** i) Using indirect proof. Derive $P \rightarrow \sim S$ from $P \rightarrow Q \vee R, Q \rightarrow \sim P, S \rightarrow \sim R, P$.

ii) Show that $R \rightarrow (S \rightarrow Q), \sim P \vee R$ and $S \Rightarrow P \rightarrow Q$

### UNIT - II

**4.** i) What is mathematical induction? Use mathematical induction to prove that 1.1! + 2.2!+...+n.n!=(n+1)!-1, wherever n is a positive integer.

ii) Write about Recursively Defined Function in detail and Give a recursive definition of $\sum_{K=0}^{n} a_K$

**(OR)**

**5.** i) Using structural induction, Show that whenever $n \geq 3, f_n > a^{n-2}$, Where $a = (1+\sqrt{5})/2$.

ii) Give a recursive algorithm for computing $a^n$, where a is a nonzero real number and n is a nonnegative

### UNIT - III

**6.** i) Write about Pigeonhole Principle in detail? Prove that if N objects are placed into K boxes, then there is at least one box containing at least $\left\lceil \frac{N}{K} \right\rceil$ objects.?

ii) Define Permutation and show in how many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

**(OR)**

7. i) Let n and r be nonnegative integers with $r \leq n$. Then prove that $C(n, r) = C(n, n - r)$ and show that in how many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

ii) What is the expansion of $(x + y)^4$ ?

## UNIT - IV

8. i) Prove that, if $F_n$ is the $n^{th}$ Fibonacci number, then $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1+\sqrt{5}}{2}\right)^{n+1}\right]$ for all integers $n \geq 0$.

ii) Solve the recurrence relation $a_n - a_{n-1} - 12a_{n-2} = 0, a_0 = 0, a_1 = 1$.

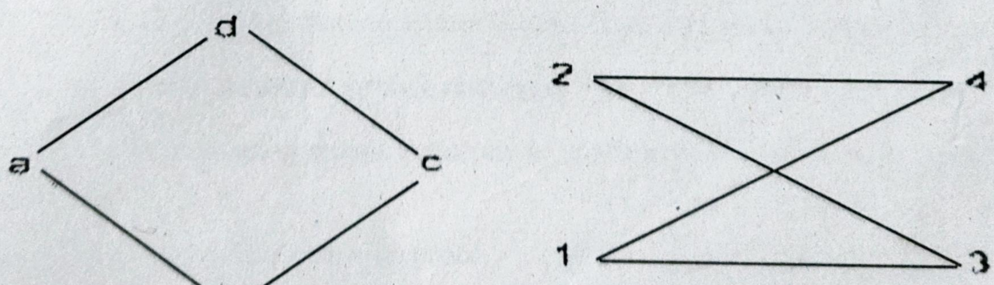
**(OR)**

9. i) Discuss about Principle of Inclusion - Exclusion and show that how many positive integers are not exceeding 1000 are divisible by 7 or 11?

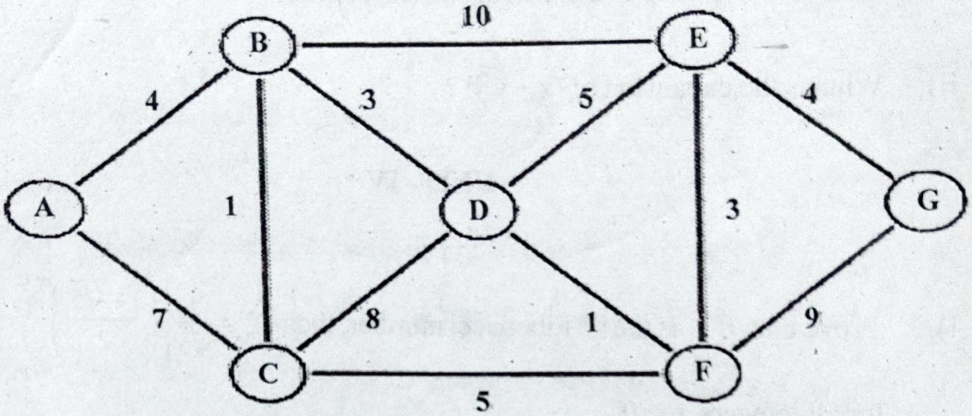
ii) Solve the recurrence relation $a_k = 3a_{k-1}$ for $k = 1,2,3,...$ and initial condition $a_0 = 2$.

## UNIT - V

10. i) Show that the graphs $G = (V, E)$ and $H = (W, F)$ shown in below figure are isomorphic.



ii) Use Dijkstra's algorithm to find the length of a shortest path between the vertices A and G in the weighted graph shown below:




(OR)

11. i) Show that $K_n$ has a Hamilton circuit whenever $n \geq 3$.

ii) Write about Graph Coloring and show how can the final exams at a university be scheduled so that no student has two exams at the same time?

# MASTER OF COMPUTER APPLICATIONS DEGREE EXAMINATION - APRIL/MAY - 2023

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### PART-A

**(Compulsory)**

**Answer any FIVE of the following questions. Each question carries 2 marks.**

**(5×2=10)**

1. a) Over the Universe of animals, Let P(x): x is whale; Q(x): x is a fish; R(x):x lives in a water. Translate the following into English.

$\exists x(\neg R(x)$

$\exists x(Q(x)\wedge\neg P(x)$

$\forall x(P(x)\wedge R(x))\rightarrow q(x)$

b) Define conjunctive and disjunctive normal forms.

c) Show that $(P\rightarrow Q)\wedge(P\vee Q)$ is a contingency

d) What is meant by strong induction?

e) How many committees of five with given chairperson can be selected from 12 persons?

f) Find the co-efficient of $x^{10}$ in $\frac{(x^3-5x)}{(1-x)^3}$

g) Solve $a_k=3a_{k-1,}$ for $k\geq 1$ with $a_0$=2

h) Define planar graph and bi-partite graph.

i) Define in - degree and out degree of a directed graph.

j) What is meant by Handshaking property of a graph?

## PART-B

**Answer any ONE full question from each Unit. Each question carries 12 mark**

**(5×12=60)**

### UNIT -I

2. a) Show that $R \to S$ can be drawn from the premises $P \to (Q \to S), \neg R \vee P$ and Q.

   b) Verify that $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \equiv [(p \to q) \wedge (q \to r) \wedge (r \to p)]$, for Primitive statements p, q, r.

**(OR)**

3. a) Prove the following by using a "direct proof":

   i) If m is even integer, then prove m +7 is odd

   ii) If k and 1 are odd integers, prove that k + 1 is even.

   b) Prove that the following argument is valid:

$$\begin{array}{l} \forall x[p(x) \to q(x)] \\ \underline{\forall x[q(x) \to r(x)]} \\ \therefore \forall x[p(x) \to r(x)] \end{array}$$

### UNIT -II

4. a) Show that $3^{4n+2} + 5^{2n+1}$ is multiple of 14 for all positive integral value of 'n' Including zero by mathematical induction.

   b) Give a recursive algorithm for computing the greatest common divisor of two Nonnegative integers a and b with $a < b$.

**(OR)**

5. a) Use the well-ordering property to prove the division algorithm.

   b) Write a recursive algorithm for computing $b^n$ mod m, where b,n, and m are integers with $m \geq 2$, $n \geq 0$, and $1 \leq b < m$.

### UNIT -III

6. a) Find the co-efficient of $x^{18}$ in $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + \ldots)^5$

   b) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

**(OR)**

7. There are 21 consonants and 5 vowels in the English alphabets. Consider only 8 letters with 3 different vowels and 5 different consonants.

   i) How many of such words can be formed?

   ii) How many begin with 'a' and end with 'b'?

   iii) How many contain the letters a, b, and c?

## UNIT - IV

8. A survey on sample 25 new cars being sold out at a local auto dealer was conducted to see which of three popular options Air conditioner(A), Radio (R), Power Windows (W) were already installed. The survey found 15 had Air Conditioners. 12 had Radios, and 11 had power Windows. 5 had Air Conditioner and Power Windows, 9 had Air conditioner and Radio, 4 had Radio and Power Windows. Three had all three options. Find number of cars which had:

   i) Only one of the option

   ii) at least one of the option

   iii) None of the options. Use principle of inclusion exclusion.

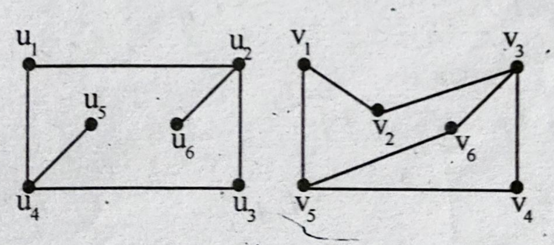
**(OR)**

9. a) Solve recurrence relation $a_n = 3\,a_{n-1} - 2\,a_{n-2}$ *for* $n \geq 2$

   b) How many integral solutions are there to $x_1+x_2+x_3+x_4+x_5=20$ where each $x_i \geq 2$?

## UNIT - V

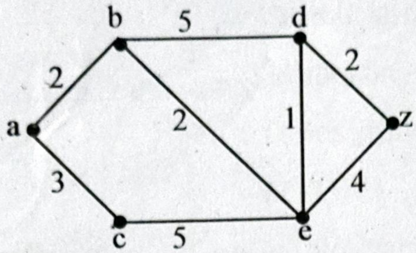
10. a) Verify the following graphs are isomorphic or not.




   b) Show that a non-empty connected graph is Eulerian if and only if all its vertices are of even degree.

**(OR)**

11. a) Find the length of a shortest path between a and z in the given weighted graph.




b) Show that "Every simple planar graph is 5-colorable.".

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