Historical & Development :- The main orign of operation Research was during 2nd world user at this time. The defence management of British has formed a Team to study the strategic and Tactical problems related to Air, land, war. Defence of the Country. Since they have limited defence resources it was necessary to decide path more effective utilization.

During the 2nd world war the military Commands are US and UK has a mission to form formulation of Specific proposals of plants stratigic and Tactical defence operations the operations research team will never participate in the war but as Advisers and significant role in winning the war with the Scintific and Systamatic Approats in the present ERA. OR has moy Areas of Applications in our Country Indian Railways, Indian Airlins, Defence organization, TATA iron sted Company, Reliance Industries, fertilizes Corporation of India, Hindustan liber Limited, Delhi clock.

Definitions :-

'OR' is a scintific method of Decision making with a Quntitative Bases in the executive departments about operations & Control

— L. MORSE & K. MBALL 1946.

2. OR is a scintific Approach of problem solving to the execbive management - H.M. WAGNOR

3. OR is an experimental and Applied science Dual teach for observing understanding and predecting the behaviour of a system to Apply in business gobvrnment society - O.R. society of America

Management Applications of OR :-

Many Areas of management has decesign making process hence various Tooles of OR can be Applied

1. finance - Budgeting & Investment :-

a) cash flow Analaysis long range Capital planing divident policy, Investment port folio Analysis

b) credit policy, Credit risk & account procedure

c) clear and compresive finance Procedure

2. purchasing of procurement and Exploration :-

(a) Rules for buying, supplies, price

(b) Bidding policy, Quautity and timing of purchase

(c) vendor selection, Replacement policy

3. Production management :-

(a) physical distribution of Products

(b) Lay out planning

(c) manufacturing procedure

(d) Maintanance & project management

4. marketing management :-

(a) product selection, Timing, Competatars

(b) sales persons, Branchures of product

(c) Advertaising, Sales promotion, Cost economics

5. HRM :-

a) HR select, wages & salaries

(b) Training & Development, Job Assignment, Job Allocation

6. Research & Development :-

(a) Trust areas of Concentration of RND

(b) project selection & Cost Analysis

(c) Reliability & Alternative design

Models of operation Research

Definition :- A model is the representation of an actual object / abstract / phototype or a situation it shows the relationship [Direct, & indirect] and interdependency of action and reaction. It is an abstract of Reality models may be classified as follows.

objective :-

The objective is to provide methods of analysis the behaviour of a system to improve the performance

1. Classification of models by structure :-

(i) Iconic model :-

They represent a situation or system, by scaling up or down [by enlarging or reducing]

Ex :- Toy, aeroplane, photographs, maps, drawings etc

This models are specific, Concrete or Comprensive descriptive, self explanatory by nature.

(ii) Analog model :-

In this model one set of Properties is applied to represents another set of properties

Ex :- Time, number, percentage, age, weight, temperature etc

(iii) Symbolic (Mathematical) models :-

utilize a set of mathematical symobols, equations, Expressions, Relations to describe a behaviour of a system. the Solution Can be derived with the well defined mathematical techniques.

2) classification of modelling in operations Research by purpose.

this models are identify by the purpose of utility they usage of the purpose may be descriptive, predictive or prescriptive.

(i) Descriptive models :-

these models will describes some aspects of a situation based on observation surveys, questionnaire result of an opinion poll represents a descriptive model

Ex :- opinion poll, exit poll, survey

(ii) predictive models :-

such models Can answer 'what if.' type of Questions, i.e they can make predictions regarding certain events

Ex :- Survey result, television network, social media answer

(iii) prescriptive models :-

when a predictive model has been repeatedly successful, it can be used to prescribe a source of action.

Ex :- operation research model, prescriptive model

3) Classification by Nature of Environment :-

(i) Deterministic model :-

Such models assume Condition of Complete certainty and perfect knowledge.

Ex :- linear programming

(ii) Probabilistic or Stochastic models :-

(These types of models usually handle such situations) They are used when the data is uncertained and doubtful (with errors)

Ex :- probability distributions, simuilation models

4) classification by Behaviour :-

(i) static models :-

The models do not Consider the impact of changes occuring during a plaining period they are Independent of decision taken over that day

(ii) Dynamic models :-

This models will Consider time as a major factor and Considers the impact of time

5) Classification by method of solution :-

(i) Analytical method :-

- This method have a specific mathematis structure and can be sloved by known analytical and mathematic technique

Ex :- All operation research models

(ii) Simulation

- they may have mathematic structure but can't be sloved by tools and techniques. A simulation model can be sloved by application of computer

⇒ phases of operation Researchers

1. formulation of the problem
2. Construction of mathematical model
3. Deriving solution from the given model
4. Testing the model (updation of the model)
5. validation of the Solution
6. Implementing the solution.

⇒ Scope of operation Research :-

Optimization techniques can be applied to the various areas.

1. In Agriculture sector :-

(a) optimum allocation of land to variety of crops as per the climatic conditions

(b) optimum distribution of water from numerous resources

2) In Finance :-

for the economic development of Country

(a) To maximize per Capita of income

(b) To derive profit plan to the organization

(c) To find best replacement policy

3) In Industry :-

optimum utilization of Various resources of the organization.

4) In Marketing Management :-

(a) Distribute the products with the minimum of Cost transportation

(b) To Maintain stock level to meet the future demand

(c) Best Adverstising Policy.

5) In HR Management :-

(a) To appoint to select sutable man power with refers to organization requirements

(b) Employee training, work load distribution, wages and salaries, economics of scale

6) In Manifacturing sector :-

(a) Quality and Quantity of products to be produce

(b) scheduling, sequence of operations, allocation of man power and machines

(c) opitimum product mix, Production plan

7) In Insurance Area of sector :-

(a) optimum premium rates for various insurances plans

(b) profit distribution with reference to insurance policies.

Various techinques of optimisation :-

1. linear programming models
2. Transporation models
3. Assign model
4. Queuing of waiting line theory models
5. game theory
6. Inventory management model
7. Dynamic Programming model
8. Network models
9. Simulation models
10. Scheduling sequencing models
11. Non linear programming model
12. Goal Programming model
13. Markovian model.

Linear Programming models (or) Linear Programming problems (LPP)

In 1947, GEORGE DANTZING and his team [working in US Airforce Departement] had observed that a large number of defence Programming and planing problems Can be formulated has optimization

[Maxinum or minimize] of a linear form or mathematical problem.

① Definition :-

The general LPP has a optimizing with the linear function of variables known as "objective function", subject to a set of linear equations inequality equations [Note as constraints / restrictions / Conditions]

General formula of LPP :-

In order to find the value of 'n' decision variables $x_1, x_2, x_3 \cdots x_n$. To maximize or minimize the objective function $z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$ and to satisfy 'm' Constraints such as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \ (\leq / = / \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \ (\leq / = / \geq)\ b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n \ (\leq / = / \geq)\ b_3.$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \cdots + a_{in}x_n \ (\leq / = / \geq)\ b_i$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$a_{m1}x + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \ (\leq / = / \geq)\ b_m$$

and also non negativity constraints or Conditions such as $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0 \cdots x_n \geq 0$

Note :-

The value of RHS Parameters $b_i\ (i = 1, 2, 3 \cdots m)$ Should be non negative (positive hole)

The $c_i\ (i = 1, 2, 3 \cdots n)$ values are objective function Co-efficients.

Matrix for of LPP :-

The LPP can be represented in the matrix form as below.

$$\text{Max } Z = C \cdot X^T$$

Subject to $A \cdot X = b \ (b \geq 0)$ and $X \geq 0$

where $X = (x_1, x_2, x_3 \text{---}, x_n, \text{---}, x_{n+m})$

$C = (c_1, c_2, c_3 \text{----} c_n, 0, \text{---} 0)$

$b = (b_1, b_2, b_3 \text{-------} b_m)$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & 1 & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & 0 & 1 & 0 & \cdots & 0 \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} & 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & 0 & 0 & 0 & 0 \cdots & 1 \end{bmatrix}$$

The vector 'x' will have all variables (decision variables stack variables, surplux variables, artifical variables).

The vector 'c' represent the corresponding coefficient of 'n' decision variables the values are zeros for other variable.

Temrminology of LPP

Solution to LPP :

Any SET $X = \{x_1, x_2, x_3 \ldots x_{n+m}\}$ of variables is known as solution to the LPP, if it satisfies the set of constraints.

Feasible Solution :-

Any SET $X = \{x_1, x_2, x_3 \ldots x_n\}$ of variables is known as feasible solution If it satisfies both Constraints set and non negative condition also

Basic Solution :-

A basic solution to the LPP, to the set of Constraints is a solution obtained by equating only 'n' variables (n+m) to the zero and solving for remaing 'm' variables, provided the determi of Coefficients of this 'm' variables is non zero

Basic vaaiable :-

The variables in the objective function that are not equat to zero are known as Basic variables.

Non - Basic variables :-

The variables in the objective function that are equated to zero are known as Non-Basic variables

Basic Feasible solution :-

A Basic feasible solution of LPP is solution to the LPP that satisfies Con set and non negativity condition.

Optimum Basic feasible solution :-

The BFS is set to the optimum if it optimi (maximize (or) minimize) the objective function.

Un-Bounded Solution :-

If the value of objective function [Z] Can be Increased or Decreased infinitly such solutions are Known as un-Bounded solutions.

In the graphical method the Convex set or Feasible region Continous infinitly (or) Indefently

In the analytical method there ~~known~~ is no living variable in the last interation or operation.

In-feasible solution :-

~~In this m~~ If the value of objective function z Can't be determined than that solution to the LPP is known as In feasible solution.

* In the graphical method there is know Common Shaded area resultant by Constraints set equation
* In the Analytical method artificial variables will be present in the last table iteration or operation

Convex set :-

The Collection of all feasible solutions to the LPP will Constitute a Convex set, who's extreme point represent basic feasible solution

Feasible region :-

that area that has a Collection of Countable number of BFS With in the feasible Solution area is known as Feasible Region.

Advantages (or) mertis of LPP :-

1. LPP helps in utilizing optimum Productive resources
2. The Quality of decision making is improving by LPP techinques
3. Modification of Solution is on Iteration basis
4. It reduses the IdLE time of operations process

Limitation (or) Demerits of LPP :-

1. In the real business problems there ~~are~~ is non-linear relationship among variables.
2. The values of decision variables are not Integers
3. They donot Consider the effect of time and uncertinity
4. large scale Complex problems Can't be solved by LPP
5. It deals with only Single objective function

Standrad from of LPP :-

1. Reverised form of objective function
2. Constraint set
3. Non-negativity

Graphical method :-

The solution to the LPP that has only two decision variables can be obtained by graphical method. The procedure steps as follows.

step 1 :- Convert every inequility constraints to a simultainous equation.

Step 2 :- find the quardinates for every constraints to represents as a line on the graph.

step 3 :- plot or Convert each equation on the graph as a stright line

step 4 :- Shade the feasible region resulting from intersection of stright lines

step 5 :- Every point on the line will satisfy the equation of that line.

step 6 :- the points present in the Common shaded region will satisfy all Constraints

step 7 :- The Common shaded region is known as feasible region.

step 8 :- Consider the extreme points of feasible region.

step 9 :- Calculate the value of objective function with these points.

problem 1 :-

Max $Z = 3x_1 + 5x_2$ subject to $x_1 + 2x_2 \leq 20$, $x_1 + x_2 \leq 15$
and $x_1 \geq 0$, $x_2 \geq 0$

Sol :-
By Graphical method

Consider the Constraints and Convert then to equation

Consider the first Constraint

Convert it into equation ;

$x_1 + 2x_2 \leq 20$

$\Rightarrow x_1 + 2x_2 = 20$

Put $x_1 = 0$ then $2x_2 = 20 \Rightarrow x_2 = 10$

The ~~Quar~~ Co-ordinates of vertex point are $(0, 10)$

Put $x_2 = 0$ then $x_1 = 20$

the Co-ordinates of vertex point are $(20, 0)$

Consider the second Constraint

Convert it into equation

$3(5) + 5(10)$
$15 + 50$
65

$x_1 + x_2 \leq 15$

$3(6)$
$18 + 50$

$\Rightarrow x_1 + x_2 = 15$

Put $x_1 = 0$ then $x_2 = 15$

The Co-ordinates of vertex point are $(0, 15)$

Put $x_2 = 0$ then $x_1 = 15$

the Co-ordinates of vertex point are $(15, 0)$

Sl No	Co-ordinates of vertex point of feasible region	Value of Z
1	A $(0, 0)$	$Z_A = 0$
2.	B $(20, 0)$	$Z_B = 60$
3.	C $(6, 10)$	$Z_C = 68$ (65)
4.	D $(0, 10)$	$Z_D = 50$

Conclusion :-

By the graphical method the value of z is 68

The optimum value of decision variables $x_1 = 6$ $x_2 = 10$

Problem 2 :-

$\max Z = x_1 + x_2$ subject to $x_1 + 2x_2 \leq 20$

$x_1 + x_2 \leq 15$, $x_2 \leq 6$ and $x_1 \geq 0$, $x_2 \geq 0$

Sol :-

By Graphical method

Consider the Constraints and Convert then to equation

Consider the first Constraint

Convert it into equation

$x_1 + 2x_2 \leq 20$

$\Rightarrow x_1 + 2x_2 = 20$

put $x_1 = 0 \Rightarrow$ then $2x_2 = 20 \Rightarrow x_2 = 10$

The Co-ordinates of vertex point are $(0, 10)$

put $x_2 = 0$, then $x_1 = 20$

The Coordinate of vertex point are $(20, 0)$

Consider the second Constraint

Convert ~~t~~ the equation

$x_1 + x_2 = 15$

put $x_1 = 0 \Rightarrow$ then $x_2 = 15$

The coordinates of vertex point are $(0, 15)$

put $x_2 = 0 \Rightarrow$ then $x_1 = 15$

The co-ordinates of vertex point are $(15, 0)$

Consider the third Constraint it Can be writen as

$\Rightarrow x_2 = 6$, $x_1 = 0$ then

The co-ordinates of vertex point for feasible region is $(0, 6)$

Set No	Coordinates of vertex point of Common Shaded Area	Value of z
1	A $(0, 0)$	$Z_A = 0$
2	B $(15, 0)$	$Z_B = 15$
3	C $(10, 5)$	$Z_C = 15$
4	D $(8, 6)$	$Z_D = 14$
5	E $(0, 6)$	$Z_E = 6$

Conclusion :-

from the graphical method the value of z is '15' [Here there are two points] The optimum values of variables (decision variables $x_1 = 15$ $x_2 = 0$. The optimum values of decision variables $x_1 = 10$ $x_2 = 5$.

Problem 3 :-

$\max z = 8x_1 + 7x_2$ subject to $3x_1 + x_2 \le 6$, $x_1 + x_2 \le 4$, $x_1 \le 4$, $x_2 \le 2$ and $x_1 \ge 0$, $x_2 \ge 0$

Problem 4 :-

$\max z = 0.5x_1 + 0.1x_2$ subject to $2x_1 + 5x_2 \le 40$, $x_1 + x_2 \le 10$ $x_2 = 6$ and $x_1 \ge 0$, $x_2 \ge 0$

Problem 5 :-

$\text{Min } Z = 1.5x_1 - 2.5x_2$ s/to $x_1 + 3x_2 \ge 3$, $x_1 + x_2 \ge 2$ and $x_1 \ge 0$; $x_2 \ge 0$

Problem 6 :-

$\text{Max } Z = 3x_1 + 2x_2$ s/to $x_1 - x_2 \le 1$, $x_1 + x_2 \ge 3$ and $x_1 \ge 0$, $x_2 \ge 0$

Sol :- UN- Bounded

Problem 7 :-

$\max z = -3x_1 + 2x_2$

s/to $x_1 + x_2 \ge 3$

$x_1 \le 5$

and $x_1 \ge 0$, $x_2 \ge 0$

Sol :- un bounded

Problem 8 :-

$\max z = 3x_1 + 2x_2$

s/to $x_1 + x_2 \le 1$

$x_1 + x_2 \ge 4$

and $x_1 \ge 0$, $x_2 \ge 0$

in feasible

Problem 9 :-

$\max Z = x_1 + x_2$

s/to $x_1 - x_2 \ge 3$

$-3x_1 + x_2 \ge 3$

and $x_1 \ge 0$, $x_2 \ge 0$ sol In-feasible

Problem 10 :-

$\min Z = 2x_1 + 4x_2$ s/to $x_1 + x_2 \ge 2$, $x_1 + 3x_2 \ge 6$ $x_1 + 2x_2 \ge 6$ and $x_1 \ge 0$, $x_2 \ge 0$

problem :- 3   8.1.14

Max $z = 8x_1 + 7x_2$ s/to $3x_1 + x_2 \le 6$ $x_1 + x_2 \le 4$

$x_1 \le 4$, $x_2 \le 2$ and $x_1 \ge 0$, $x_2 \ge 0$

By Graphical method

Consider the Constraints and Convert than to equation

Consider the first Constraint

Convert it into equation

$$3x_1 + x_2 = 6$$

put $x_1 = 0 \Rightarrow$ then $x_2 = 6$

The Cordinates of vertex point are $(0, 6)$

put $x_2 = 0 \Rightarrow$ then $3x_1 = 6 \Rightarrow x_1 = 2$

The Coordinates of vertex point are $(2, 0)$

Consider the second Constraint

Convert it into equation

$$x_1 + x_2 = 4$$

put $x_1 = 0 \Rightarrow$ then $x_2 = 4$

The Cordinates of vertex point are $(0, 4)$

put $x_2 = 0 \Rightarrow$ then $x_1 = 4$

the Coordinates of vertex point are $(4, 0)$

Consider the third constraint it Can be written as

$\Rightarrow x_1 = 4$, $x_2 = 0$

the Coordinates of vertex point are $(4, 0)$

Consider the fourth Constraint it Can be written as

$\Rightarrow x_2 = 2$, $x_1 = 0$

the Coordinates of vertex point are $(0, 2)$

Sr. No	Co-ordinates of vertex point of Common shaded Area	value of z
1	A (0,0)	$Z_A = 0$
2	B (2,0)	$Z_B = 16$
3	C (1,2)	$Z_C = 22$
4	D (0,2)	$Z_D = 14$

Conclusion :- By the graphical method the value of z is 22 the optimum value of decision Variables $x_1 = 1$, $x_2 = 2$.

Problem 4 :-

$\max z = 0.5x_1 + 0.1x_2$ subject to $2x_1 + 5x_2 \le 40$, $x_1 + x_2 \le 10$, $x_2 = 6$ and $x_1 \ge 0$, $x_2 \ge 0$

By Graphical method

Consider the Constraints and Convert then to equation.

Consider the first Constraint and Convert into equation

$\Rightarrow 2x_1 + 5x_2 = 40$

put $x_1 = 0 \Rightarrow$ then $5x_2 = 40 \Rightarrow x_2 = 8$

the co-ordinates of vertex point are (0, 8)

put $x_2 = 0 \Rightarrow$ then $2x_1 = 40 \Rightarrow x_1 = 20$

the co-ordinates of vertex point are (20, 0).

Consider the second equation & Convert it the equation

$\Rightarrow x_1 + x_2 = 10$

put $x_1 = 0 \Rightarrow$ then $x_2 = 10$

the Co-ordinates of vertex Point are (0, 10)

put $x_2 = 0 \Rightarrow$ the $x_1 = 10$

the Co-ordinates of vertex point are (10, 0)

consider the third Constraint it can be written as

$\Rightarrow x_2 = 6$ (0, 6)

set NO	Coordinates vertex point of Common Shaded region Area	value of z
1	A (0,0)	$Z_A = 0$
2	B (10,0)	$Z_B = 5$
3	C (4,6)	$Z_C = 2.6$
4	D (0,6)	$Z_D = 0.6$

Conclusion :-

By the graphical method the value of z is '5' the optimum value of decision variables $x_1 = 10$ & $x_2 = 0$

problem :- 11

$\text{MIN } z = 12x_1 + 16x_2$

S|to $3x_1 + 4x_2 \geq 24$

$2x_1 + x_2 \geq 10$

$5x_1 + 3x_2 \geq 30$

and $x_1 \geq 0, x_2 \geq 0$

problem 12 :-

$\text{Max } z = 6x + 8y$

S|to $2x + 3y \geq 16$

$4x + 2y \geq 16$

and $x \geq 0, y \geq 0$

sol unbounded

problem 13 :-

$\text{Max } z = 5x_1 + 3x_2$

S|to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

and $x_1 \geq 0, x_2 \geq 0$

problem :- 5 $\text{Min } z = 1.5x_1 - 2.5x_2$ S|to $x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$ and $x_1 \geq 0, x_2 \geq 0$

By Graphical method

Consider the Constraints Convert them into Simultaneous equations

Consider the first Constraints it Can be written as

$x_1 + 3x_2 = 3$

put $x_1 = 0$ then $3x_2 = 3 \Rightarrow x_2 = 1$

Coordinates of vertex point of feasible region are (0,1)

put $x_2 = 0$ then $x_1 = 3$

the Coordinates of vertex point of feasible region are (3,0)

Consider the second Constraint it can be written as

$x_1 + x_2 = 2$

Put $x_1 = 0$ then $x_2 = 2$
the co-ordinates of vertex point of the feasible region are
$(0, 2)$

put $x_2 = 0$ then $x_1 = 2$
the coordinates of vertex point of the feasible region are
$(2, 0)$

set No	Coordinates vertex point of Common shaded region Area	value of Z
1	A $(0, 2)$	$Z_A = -5$
2	B $(1.5, 0.5)$	$Z_B = 1$
3	C $(3, 0)$	$Z_C = 4.5$

Conclusion :- from the graphical method the value of 'z' is '1' the optimum value of decision Variables $x_1 = 1.5$ & $x_2 = 0.5$

Problem :-6
$\text{Max } z = 3x_1 + 2x_2$ s/to $x_1 - x_2 \le 1$, $x_1 + x_2 \ge 3$ and $x_1 \ge 0, x_2 \ge 0$
By Graphical method
Consider the Constraints convert them into simultaneous equations
Consider the first Constraints it can be written as
$x_1 - x_2 = 1$
put $x_1 = 0$ then $-x_2 = 1$ and $x_2 = -1$
Coordinates of Vertex point of feasible region are $(0, -1)$
put $x_2 = 0$ then $x_1 = 1$
Coordinates of vertex point of feasible region are $(1, 0)$
Consider the second Constraint which can be written as
$x_1 + x_2 = 3$
put $x_1 = 0$ then $x_2 = 3$
Coordinates of vertex point of feasible region is $(0, 3)$
put $x_2 = 0$ then $x_1 = 3$
Coordinates of vertex point of feasible region is $(3, 0)$

By the graphical method the resultant feasible region Continuous to be indefenitly or infinitly

the Solution to the given problem is unbounded

Problem :- 8 Max $Z = 3x_1 - 2x_2$ s/to $x_1 + x_2 \leq 1$, $x_1 + x_2 \geq 4$
$x_1 \geq 0$, $x_2 \geq 0$

By graphical method

Consider the Constraints they Can be Converted into equations

Consider the first Constraint it Can be written as

$x_1 + x_2 \leq 1$

put $x_1 = 0$ then $x_2 = 1$

the coordinates of vertex point of feasible region is $(0, 1)$

put $x_2 = 0$ then $x_1 = 1$

the Coordinates of vertex point of feasible region is $(0, 1)$

Consider the Second Constraint

$x_1 + x_2 = 4$

put $x_1 = 0$ then $x_2 = 4$

the Coordinates of vertex point of feasible region is $(0, 4)$

put $x_2 = 0$ then $x_1 = 4$

the Coordinates of vertex point of feasible region is $(4, 0)$

Conclusion :-

from the graphical method there is no Common Shaded area

Hence the Solution for the given problem is In feasible.

problem 7 :- $\max z = -3x_1 + 2x_2$ S/to $x_1 + x_2 \geq 3$ $x_1 \leq 5$ & $x_1 \geq 0$ $x_2 \geq 0$

By graphical method

Consider the Constraints they Can be Converted into equation

Consider the first Constraint it Can be written as

$x_1 + x_2 = 3$ put $x_1 = 0$ then $x_2 = 3$

the coordinates of vertex point of feasible region is $(0, 3)$

[the Consider] put $x_2 = 0$ then $x_1 = 3$

the coordinates of vertex point of feasible region is $(3, 0)$

Consider the Second Constraint it Can be written as

$x_1 = 5$ & $x_2 = 0$

the coordinates of vertex point of feasible region is $(5, 0)$

Conclusion :-

By the graphical method the resultant feasible region Continuous to be indefenitly or infinitly

The solution to the given problem is unbounded

problem 9 :- $\max z = x_1 + x_2$ S/to $x_1 - x_2 \geq 3$, $-3x_1 + x_2 \geq 3$ and $x_1 \geq 0$, $x_2 \geq 0$ &

By graphical method

Consider the Constraints they Can be Converted into equations

Consider the first Constraint it Can be written as

$x_1 - x_2 = 3$

put $x_1 = 0$ then $-x_2 = 3 \Rightarrow x_2 = -3$

the Coordinates of vertex point of feasible region is $(0, -3)$

put $x_2 = 0$ then $x_1 = 3$ the coordinates of vertex are $(3, 0)$

Consider the Second Contraint it Can be written as

$-3x_1 + x_2 = 3$

put $x_2 = 0$ then $-3x_1 = 3 \Rightarrow -x_1 = 1 \Rightarrow x_1 = -1$

the Coordinates of vertex point of feasible region is $(-1, 0)$

put $x_1 = 0$ then $x_2 = 3$

the coordinates of vertex point of feasible region is $(0, 3)$

from the graphical method there is no Common shaded area
Hence the Solution for the given problem is Infeasible.

10) $\min z = 2x_1 + 4x_2$ s/to $x_1 + 2x_2 \geq 2$ $x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \geq 6$ and $x_1 \geq 0$ $x_2 \geq 0$

By graphical method
Consider the constraints they can be converted into equation
Consider the first Constraint it can written as

$x_1 + 2x_2 = 2$

put $x_1 = 0$ then $2x_2 = 2 \Rightarrow x_2 = 1$
Coordinates of vertex point of the feasible region is
$(0, 1)$
put $x_2 = 0$ then $x_1 = 2$
Coordinates of vertex point of the feasible region is $(2, 0)$

Consider the second Constraint

$x_1 + 3x_2 = 6$

put $x_1 = 0$ then $\cancel{3}x_2 = \cancel{6}^{2} \Rightarrow x_2 = 2$
Coordinates of vertex point of the feasible region is
$(0, 2)$
put $x_2 = 0$ then $x_1 = 6$
Coordinates of vertex point of the feasible region is
$(6, 0)$

Consider the third Constraint

$x_1 + 2x_2 = 6$

put $x_1 = 0$ then $2x_2 = 6 \Rightarrow x_2 = 3$
Coordinates of vertex point of the feasible region is $(0, 3)$
put $x_2 = 0$ then $x_1 = 6$
Coordinates of vertex point of the feasible region $(6, 0)$

Set No	Coordinates vertex point of Common Shaded region Area	Value of z
1	A $(0,0)$	$z_A = 0$
2	B $(6,0)$	$z_B = 12$
3	C $(0,3)$	$z_C = 12$
~~4~~	~~E~~ (-)	~~z_E~~
5		z -

$z = 12 \quad (x_1, x_2) = (0, 3)$

$(x_1, x_2) = (0, 6)$

11\. $MIN_z = 12x_1 + 16x_2$ S.to $3x_1 + 4x_2 \geq 24$; $2x_1 + x_2 \geq 10$, $5x_1 + 3x_2 \geq 30$ and $x_1 \geq 0$, $x_2 \geq 0$

By graphical method

Consider the Constraints and Convert them to equations

Consider the first Constraint $3x_1 + 4x_2 = 24$

put $x_1 = 0$ then $4x_2 = 24 \Rightarrow x_2 = 6$

Coordinates of vertex point of the feasiable region is $(0, 6)$.

Consider $x_2 = 0$ then $3x_1 = 24 \Rightarrow x_1 = 8$

Coordinates of vertex point of the feasiable region is $(8, 0)$

Consider the Second Constraint. $2x_1 + x_2 = 10$

put $x_1 = 0$ then $x_2 = 10$

Coordinates of vertex point of the feasiable region is $(0, 10)$

put $x_2 = 0$ then $2x_1 = 10 \Rightarrow x_1 = 5$

Coordinates of vertex point of the feasiable region is $(5, 0)$

Consider the Third Constraint $5x_1 + 3x_2 = 30$

put $x_1 = 0$ then $3x_2 = 30 \Rightarrow x_2 = 10$

Coordinates of vertex point of the feasiable region is $(0, 10)$

put $x_2 = 0$ then $5x_1 = 30 \Rightarrow x_1 = 6$

Coordinates of vertex point of the feasiable region is $(6, 0)$

set	Coordinates of vertex point of Common shaded	Value z
1	A (0, 10)	$z_A = 160$
2	B (4·4, 2·7)	$z_B = 96$
3	C (8, 0)	$z_C = 96$

problem :- 12

max $z = 6x + 8y$ s/to $2x + 3y \geq 16$ $4x + 2y \geq 16$ & $x \geq 0$ $y \geq 0$

By graphical method

Consider the Constraints they Can be Converted into equation

Consider the first Constraint

$$2x + 3y = 16$$

put $x = 0$ then $3y = 16 \Rightarrow y = \frac{16}{3} = 5{\cdot}3$

Coordinates of vertex point of the feasible region is

$\cancel{(0, \frac{16}{3})}$ (0, 5·3)

put $y = 0$ then $2x = 16 \Rightarrow x = 8$

Coordinates of vertex point of the feasible region is

(8, 0)

1	A(0,8)	$z_A = 64$
2	B(2,4)	$z_B = 44$
3	C(8,0)	$z_C = 48$

Consider the second Constraint

$$4x + 2y = 16$$

put $x = 0$ then $2y = 16 \Rightarrow y = 8$

Coordinates of vertex point of the feasible region is

(0, 8)

put $y = 0$ then $4x = 16 \Rightarrow x = 4$

Coordinates of vertex point of the feasible region is

(4, 0)

By the graphical method the resultant feasible region Continuous to be indefenitly (or) infinitly
the Solution to the given problem is unbounded

13) Max $z = 5x_1 + 3x_2$ s/to $3x_1 + 5x_2 \leq 15$, $5x_1 + 2x_2 \leq 10$
and $x_1 \geq 0$, $x_2 \geq 0$

By graphical method
Consider the Constraints they Can be converted into equations
Consider the first Constraint

$$3x_1 + 5x_2 = 15$$

Put $x_1 = 0$ then $5x_2 = 15 \Rightarrow x_2 = 3$
Coordinates of vertex point of the feasible region are $(0, 3)$
put $x_2 = 0$ then $3x_1 = 15 \Rightarrow x_1 = 5$
Coordinates of vertex point of the feasible region are $(5, 0)$
Consider the Second Constraint

$$5x_1 + 2x_2 = 10$$

Put $x_1 = 0$ then $2x_2 = 10 \Rightarrow x_2 = 5$
Coordinates of vertex point of the feasible region are $(0, 5)$
Put $x_2 = 0$ then $5x_1 = 10 \Rightarrow x_1 = 2$
Coordinates of vertex point of the feasible region are $(2, 0)$

Set No	Coordinates of vertex point of feasible region	value of z
1	A (0, 0)	$z_A = 0$
2	B (2, 0)	$z_B = 10$
3	C (1, 2.4)	$z_C = 12.2$
4	D (0, 3)	$z_D = 9$

Conclusion :-
from the graphical method, the value of z is 12.2. the optimum values of Decission variables $x_1 = 1$, $x_2 = 2.4$

Analytical method :-

Simplex method :-

The analytical method LPP is simplex method it is an algorithm [A rule of procedure, involving repetitive application of a specified procedure]. It is based on fundamental therom of linear programming.

→ This method provides a systematic algorithm that starts (or) begins from a basic feasible solution to another improved solution in a prescribed method such that the value of objective function is improved step wise.

Scack, Surplus & Artificial variables :-

Slack variable :-

If the Contraint Equation has a symbol "≤" in the equation, it can be converted into simultaneous equation by adding a non-negative value [positive quantity to the LHS of the Contraint equation

Ex :- $2x_1 + 3x_2 \le 4$

$\Rightarrow 2x_1 + 3x_2 + S = 4$

where $x_1 \ge 0$, $x_2 \ge 0$

and $S \ge 0$

$2x_1 + 3x_2 \le 4$

$3x_1 + 4x_2 \le 5$

$\Rightarrow 2x_1 + 3x_2 + S_1 = 4$

$3x_1 + 4x_2 + S_2 = 5$

where $x_1 \ge 0$, $x_2 \ge 0$,

$S_1 \ge 0$, $S_2 \ge 0$

Surplus Variables :-

If the Constraint equation of LPP has a symbol "$\geq$" in its equation, it can be Converted into simultaneous linear equation by substracting a non negative value (positive quantity from the LHS of that Constraint equation)

Ex :-

$2x_1 + 3x_2 \geq 4$

$\Rightarrow 2x_1 + 3x_2 - S = 4$

and $x_1 \geq 0$, $x_2 \geq 0$

& $S \geq 0$

$2x_1 + 3x_2 \geq 4$

$3x_1 + 4x_2 \geq 5$

$\Rightarrow 2x_1 + 3x_2 - S_1 = 4$

$3x_1 + 4x_2 - S_2 = 5$

and $x_1 \geq 0$, $x_2 \geq 0$

$S_1 \geq 0$ & $S_2 \geq 0$

Artificial variables :-

In the LPP if a Constraint has "$\geq$" And (or) "$=$", a non-negative values (positive quantity) will be added to the LHS of the Constraint equation they are known as Artifical variables. they are momentary ficituos (temporary)

→ They donot have any physical meaning.

→ It is useful to begin will Initial basic feasible Solution

Ex :- $2x_1 + 3x_2 \geq 4$

$\Rightarrow 2x_1 + 3x_2 - S + A = 4$

where $x_1 \geq 0$, $x_2 \geq 0$

$S \geq 0$, $A \geq 0$

$2x_1 + 3x_2 \geq 4$

$\Rightarrow 2x_1 + 3x_2 - S_1 + A_1 = 4$

$3x_1 + 4x_2 \geq 5$

$\Rightarrow 3x_1 + 4x_2 - S_2 + A_2 = 5$

where $x_1 \geq 0$, $x_2 \geq 0$, $S_1 \geq 0$, $S_2 \geq 0$

& $A_1 \geq 0$, $A_2 \geq 0$

Standrad form of LPP :-

The Characteristics of standrad form of LPP

1. All the Contraints should be Converted into equations and also non-negativity Constraints.
2. The [RHS] Right hand side parameter should be non-negative (positive)

3. All variables should be non-negative
(Decission, variable, slack variable, surplus variable and Artifical variable ≥ 0)
4. The objective function should be in appropriate form
5. The Coefficients of variable should be correct form in the objective function

Note :-

$(0)S_1 + (0)S_2$

The Coefficient of slack and ~~sup~~ surplus variables in the objective function is taken as 'zero'. Hence, the conversion of the constraints to simultaneous equation will not affect the value of objective.

problem : 1 $\quad Max\ Z = 3x_1 + 2x_2$ S/to $x_1 + x_2 \leq 4$, $x_1 - x_2 \leq 2$ and $x_1 \geq 0$, $x_2 \geq 0$

sol :-
By Simplex method
By Taha method

Check, if the value of RHS of Constrain set is Non-negative.

Consider the Constraints convert them into equations.
Consider the first Constraint
It can be written as

$$x_1 + x_2 \leq 4$$

$$\Rightarrow x_1 + x_2 + S_1 = 4$$

Consider the second Constraint. It can be written as

$$x_1 - x_2 \leq 2$$

$$\Rightarrow x_1 - x_2 + S_2 = 2$$

where $x_1 \geq 0$, $x_2 \geq 0$, $S_1 \geq 0$, $S_2 \geq 0$

Consider the objective function

It Can written as

$Z = 3x_1 + 2x_2$

$\Rightarrow \quad Z = 3x_1 + 2x_2 + (0)S_1 + (0)S_2$

$Z = -3x_1 - 2x_2 - (0)S_1 - (0)S_2 = 0$

Initialization :-

put $x_1 = x_2 = 0$ then $S_1 = 4$ & $S_2 = 2$

where x_1 & x_2 are Decission variable & Non basic

S_1 & S_2 are Slack variable & basic

Table 1 :- [Iteration]

Basic Variable	Coefficiens of variables x_1	x_2	S_1	S_2	RHS value (or) Solution value	Ratio Value
Z	-3	-2	0	0	0	-
S_1	1	1	1	0	4	$\frac{4}{1} = 4$
S_2	1	-1	0	1	2	$\frac{2}{1} = 2$

Key Column

key element

Key row

for Maximization :-

[The objective function has maximization]

Consider "most negative value" (The coefficient of negative sign (-ve) must be maximum)

* Neglet +ve & 0 for maximumization

Consider Least positive (+ve) value in the ratio Column

$$\text{Ratio value} = \frac{\text{RHS values}}{\text{Key column values}}$$

* Neglet $\frac{0}{0}$, 0, ∞, -ve $(\infty = \frac{1}{0})$

Note:-

[The Intersection of key row & key colum is key element]

Table :-2

Basic variable	Coefficient of variable x_1	x_2	s_1	s_2	RHS value (or) Solution value	Ratio value
Z	0	-5	0	3	6	—
uneffect variable S_1	0	$\boxed{2}$	1	-1	2	$\frac{2}{2} = 1$ → leaving position
pivot row → x_1	1	-1	0	1	2	-ve $\left[\frac{2}{-1} = -2\right]$

key element   key column   key row

'Z' has most negative go to next table

$$\text{PIVOT Row Values} = \frac{\text{key row values}}{\text{key element}}$$

$$\text{New Row values} = \text{Old Row values} - \left[\text{Corresponding Key Colum Value} * \text{PIVOT Row Value}\right]$$

Z - Row

L.H.S
$-3 - [-3 \times 1] = 0$
$-2 - [-3 \times -1] = -5$
$0 - [-3 \times 0] = 0$
$0 - [-3 \times 1] = 3$
RHS $0 - [-3 \times 2] = 6$

S - row

$1 - [1 \times 1] = 0$
$1 - [1 \times -1] = 2$
$1 - [1 \times 0] = 1$
$0 - [1 \times 1] = -1$
$4 - [1 \times 2] = 2$

Table 3. (S_1 leaving x_2 entering)

Basic variable	Coefficient of variable x_1	x_2	s_1	s_2	RHS Value (or) Solution Value	Ratio value
Z	0	0	$5/2$	$\frac{1}{2}$	11	—
x_2	0	1	$1/2$	$-1/2$	1	
pivot row x_1	1	0	$\frac{1}{2}$	$\frac{1}{2}$	3	

Z - Row $\qquad$ x - row

$0 - [-5 \times 0] = 0$ $\qquad$ $1 - [-1 \times 0] = 0$

$-5 - [-5 \times 1] = 0$ $\qquad$ $-1 - [-1 \times 1] = 0$

$0 - [-5 \times \frac{1}{2}] = \frac{5}{2}$ $\qquad$ $0 - [-1 \times \frac{1}{2}] = \frac{1}{2}$

$3 - [-5 \times -\frac{1}{2}] = \frac{1}{2}$ $\qquad$ $1 - [-1 \times -\frac{1}{2}] = \frac{1}{2}$ $\qquad$ $1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

$6 - [-5 \times 1] = 11$ $\qquad$ $2 - [-1 \times 1] = 2 + 1 = 3$

Conclusion :- The Iteration stop there is no entering variable into the solution

Hence, it is the "OPTIMAL TABLE"

from the last iteration, the value of $Z = 11$ and $x_1 = 3$ $x_2 = 1$

Cross check :- $\text{Max } z = 3x_1 + 2x_1$

$= 3(3) + 2(1)$

$= 9 + 2$

$= 11$

$3 - [-5 \times -\frac{1}{2}]$

$3 \{ \frac{5}{2} \}$

$\frac{6-5}{2} = \frac{1}{2}$

BIG - M method :-

Procedural Steps as flows :-

1. Convert the given problem into Standrad form
2. Add non negative surplus and artifical variables to LHSR the Constraint equation having the symbols has "$\geq$" (or) "$=$"
3. when artificial variables are added it violets the Solution of the problem
4. This Can be remove by assiging zero values to the artificial variables in the final Solution [final table or iteration]

5. This can be archivied by assiging very large values [also known as unique ~~pen~~ phynymial] to this variables in the objective function.

6. Such very large values given to the objective function as follows.

"$-m$" $\rightarrow$ for maximization

"$+m$" $\rightarrow$ for minimization.

Problem 2 :-

$\text{MAX } z = 3x_1 - x_2$ S/to $2x_1 + x_2 \geq 2$, $x_1 + 3x_2 \leq 3$

$x_2 \leq 4$ and $x_1 \geq 0$, $x_2 \geq 0$

By Big-M method

By TAHA method

check whether RHS values of the Constraint equation should be positive

Consider the Constraints they can be converted into simulteonous linear equation

Consider the first Constrain $[2x_1 + x_2 \geq 2]$.

It can be written as

$$\Rightarrow 2x_1 + x_2 - S_1 + A_1 = 2$$

Consider the second Constrain it can be written as

$$x_1 + 3x_2 \leq 3$$

$\Rightarrow x_1 + 3x_2 + s_2 = 3$

Consider the third Constrain it Can be written as

$x_2 \leq 4$

$\Rightarrow x_2 + s_3 = 4$

where $x_1 \geq 0,\ x_2 \geq 0,\ s_1 \geq 0,\ s_2 \geq 0,\ s_3 \geq 0\ \&\ A_1 \geq 0$

Initialization :-

put $x_1 = x_2 = s_1 = 0$ then

$A_1 = 2,\ s_2 = 3,\ s_3 = 4$

wheae x_1 & x_2 are decision variable & Non Basic

$s_1 \rightarrow$ surplus variable & non-basic

s_2 & $s_3 \rightarrow$ slack variable & basic

$A_1 \rightarrow$ Artificial variable & Basic

Consider the objection function it Can be written as

$z = 3x_1 - x_2$

$z = 3x_1 - x_2 + (0)s_1 + (0)s_2 + (0)s_3 - mA_1$

$= 3x_1 - x_2 - m(2 - 2x_1 - x_2 + s_1)$

$= 3x_1 - x_2 - 2m + 2mx_1 + mx_2 - ms_1$

$= 3x_1 - x_2 + 2mx_1 + mx_2 - ms_1 - 2m$ $\quad -x_2(1-m)$

$= x_1(2m+3) + x_2(m-1) - ms_1 - 2m$

$z - x_1(2m+3) - x_2(m-1) + ms_1 = -2m$

entering variable

Table 1 :-

Basic variable	x_1	x_2	s_1	s_2	s_3	A_1	RHS value (or) Solution Value	Ratio Value
Z	$-(2m+3)$	$-(m-1)$	m	0	0	0	$-2m$	–
s_2	1	3	0	1	0	0	3	$3/1 = 3$
s_3	0	1	0	0	1	0	4	$\frac{4}{0} = \infty$
A_1	2	1	-1	0	0	1	2	$\frac{2}{2} = 1$

least value

Table :- 2

Basic variable	Coefficient of variable x_1	x_2	S_1	S_2	S_3	A_1	RHS value (or) Solution value	Ratio value
Z	0	$5/2$	$3/2$	0	0	$\left(\frac{2m+3}{2}\right)$	3	—
S_2	0	$5/2$	$1/2$	1	0	$\frac{-1}{2}$	2	
S_3	0	1	0	0	1	0	4	
pivot row → x_1	1	$1/2$	$-1/2$	0	0	$1/2$	1	

Since there is no entering variable in the Solution. the iteration stops

Hence, it is "optimal table"

from the analytical method value of $Z = 3$

the optimum values of variables is decission variables

$x_1 = 1$

$x_2 = 0$

$A_1 = 0$

$\therefore \ Z = 3x_1 - x_2$

$Z = 3(1) - 0$

$Z = 3$

problem 3 :— $\min \ Z = 5x_1 + 4x_2 + 3x_3$

s/to $x_1 + x_2 + x_3 = 100$

$x_1 \leq 20$

$x_2 \geq 30$

$x_3 \leq 40$ and $x_1, x_2, x_3 \geq 0$

Consider the Constraints they be written as

$$x_1 + x_2 + x_3 = 100$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + \overset{\text{Artifial value.}}{A_1} = 100$$

Consider the second Constraint

$$x_1 \leq 20$$

$$x_1 + s_2 = 20$$

Consider the third Constraint it Can be written as

$$x_2 \geq 30$$

$$x_2 - s_3 + A_3 = 30$$

Consider the fourth Constraint it Can be written as

$$x_3 \leq 40$$

$$x_3 + s_4 = 40$$

Initialization :-

put $x_1 = x_2 = x_3 = s_3 = 0$ then

$A_1 = 100$, $s_2 = 20$, $A_3 = 30$, $s_4 = 40$

$x_1, x_2, x_3, s_2, s_3, s_4, A_1, A_3 \geq 0$

$$Z = 5x_1 + 4x_2 + 3x_3 + (0)s_2 + (0)s_3 + (0)s_4 + mA_1 + mA_3$$

$$= 5x_1 + 4x_2 + 3x_3 + m(100 - x_1 - x_2 - x_3) + m(30 - x_2 + s_3)$$

$$= 5x_1 + 4x_2 + 3x_3 + 100m - mx_1 - mx_2 - mx_3 + 30m - mx_2 + ms_3$$

$$= 5x_1 + 4x_2 + 3x_3 + 130m - mx_1 - 2mx_2 - mx_3 + ms_3$$

$$Z = \cancel{2} - x_1(m-5) - x_2(2m-4) - x_3(m-3) + ms_3 + 130m$$

$$Z + x_1(m-5) + x_2(2m-4) + x_3(m-3) - ms_3 = 130m$$

Table :- 1

entering variable

Basic variable	Coefficient of variable x_1	x_2	x_3	S_2	S_3	S_4	A_1	A_3	RHS value (or) Solution value	Ratio Value	
Z	$m-5$	$2m-4$	$m-3$	0	$-m$	0	0	0	$130m$	—	
S_2	1	0	0	1	0	0	0	0	20	∞	$\frac{20}{0}$
S_4	0	0	1	0	0	1	0	0	40	∞	$\frac{40}{0}$
A_1	1	1	1	0	0	0	1	0	100	100	$\frac{100}{1}$
A_3	0	1	0	0	-1	0	0	1	30	30	$\frac{30}{1}$

key element

Table :- 2

Basic value	Coefficient of variable x_1	x_2	x_3	S_2	S_3	S_4	A_1	A_3	RHS value (or) Solution value	Ratio Value
Z	$(m-5)$	0	$(m-3)$	0	$(m-4)$	0	0	$-(2m+4)$	$70m+120$	—
S_2	1	0	0	1	0	0	0	0	20	∞
S_4	0	0	1	0	0	1	0	0	40	40
A_1	1	0	1	0	1	0	1	-1	70	70
x_2	0	1	0	0	-1	0	0	1	30	∞

pivot row

Table :- 3

Basic value	x_1	Coefficient of variable x_2	x_3	S_2	S_3	S_4	A_1	A_3	RHS value	Ratio Value
Z	$m-5$	0	0	0	$(m-4)$	$-(m-3)$	0	$-(2m-4)$	$30m+240$	—
S_2	1	0	0	1	0	0	0	0	20	$\frac{20}{1}$
x_3	0	0	1	0	0	1	0	0	40	∞
A_1	1	0	0	0	1	-1	1	-1	30	$\frac{30}{1}$
x_2	0	1	0	0	-1	0	0	1	30	∞

pivot row

Table :- 4

Basic value	x_1	x_2	x_3	S_2	S_3	S_4	A_1	A_3	RHS value	Ratio Value
Z	0	0	0	$-(m-5)$	$(m-4)$	$-(m-3)$	0	0	$10m+340$	–
x_1	1	0	0	1	0	0	0	0	20	∞
x_3	0	0	1	0	0	1	0	0	40	∞
pivot row $\Rightarrow$ A_1	1	0	0	-1	1	-1	1	-1	10	10
x_2	0	1	0	0	-1	0	0	1	30	$-ve$

Table :- 5

Basic value	x_1	x_2	x_3	S_2	S_3	S_4	A_1	A_3	RHS	Ratio
Z	0	0	0	1	0	-1	$-(m-4)$	$m-4$	380	—
x_1	1	0	0	1	0	0	0	0	20	
x_3	0	0	1	0	0	1	0	0	40	
S_3	0	0	0	-1	1	-1	1	-1	10	
x_2	0	1	0	-1	0	-1	1	0	40	

$5(20) + 4(40) + 3(40)$

$100 + 160 + 120$

100
160
120
380

Conclusion :-

Since there is no entering variable into the Solution the iteration stops

Hence, it is the "optimal table"

from the analytical method the value of $Z = 380$

The optimum values of decission variables

$x_1 = 20$, $x_2 = 40$, $x_3 = 40$, $A_1 = 0$

problem : 4 Jan 2022

Max $z = 5x_1 + 3x_2$ S/to $3x_1 + 5x_2 \leq 15$, $5x_1 + 2x_2 \leq 10$

and $x_1, x_2 \geq 0$

By simplex method

By TAHA method

Check wheather the RHS Values of the Constraints are positive

Consider the Constraints, they can be written into equations

Consider the first Constraint,

it can be written as

$$3x_1 + 5x_2 \leq 15$$

$$\Rightarrow 3x_1 + 5x_2 + S_1 = 15$$

Consider the Second Constraint,

it can be written as

$$5x_1 + 2x_2 \leq 10$$

$$\Rightarrow 5x_1 + 2x_2 + S_2 = 10$$

where $S_1 \geq 0$, $S_2 \geq 0$

Initialization :

put $x_1 = x_2 = 0$ then

$S_1 = 15$, $S_2 = 10$

where x_1, x_2 are decission variables and non-basic

S_1, S_2 are slack variables and basic variables

Consider the objective function,

it can be Converted as

$$5x_1 + 3x_2 = z$$

$$\Rightarrow z = 5x_1 + 3x_2 + (0)S_1 + (0)S_2$$

$$\Rightarrow z - 5x_1 - 3x_2 - (0)S_1 - (0)S_2 = 0$$

Table : 1

Basic variables	Coefficient of variables x_1	x_2	s_1	s_2	RHS values	Ratio
Z	-5	-3	0	0	0	–
S_1	3	5	1	0	15	$\frac{15}{3} = 5$
S_2	5	2	0	1	10	$\frac{10}{5} = 2$

Table 2 :

Basic Variables	Coefficient of variables x_1	x_2	S_1	S_2	RHS value	Ratio
Z	0	-1	0	1	10	–
S_1	0	$\frac{19}{5}$	1	$\frac{-3}{5}$	9	$\frac{45}{19}$
→ x_1	1 $\cancel{5}$	$\frac{2}{5}$	0	$\frac{1}{5}$	2 $\left[\frac{10}{5}\right]$	5

Z value :-

$-5 - [-5 \times 1] = -5 + 5 = 0$

$-3 - [-5 \times \frac{2}{5}] = -3 + \frac{10}{5} \Rightarrow \frac{-15+10}{5} \Rightarrow \frac{-8}{8} \Rightarrow -1$

$0 - [-5 \times 0] = 0$

$0 - [-5 \times \frac{1}{5}] = 1$

$0 - [-5 \times 2] = 10$

S_1 value :-

$3 - [3 \times 1] = 0$

$5 - [3 \times \frac{2}{5}] = 5 - \frac{6}{5} \Rightarrow \frac{25-6}{5} \Rightarrow \frac{19}{5}$

$1 - [3 \times 0] = 1$

$0 - [3 \times \frac{1}{5}] = -\frac{3}{5}$

$15 - [3 \times 2] = 15 - 6 = 9$

$\frac{\frac{19}{15}}{\frac{\cancel{19}}{\cancel{18}}} = 1$, $\frac{\frac{19}{5} \times 1}{\frac{5}{19}}$, $\frac{\frac{-3}{\cancel{5}}}{\frac{19}{\cancel{5}}} = \frac{-3}{19}$, $\frac{9}{\frac{19}{5}}$ $\frac{45}{19}$

Table : 3

Basic variables	x_1	x_2	S_1	S_2	RHS value	Ratio
Z	0	0	$\frac{5}{19}$	$\frac{16}{19}$	$\frac{235}{19}$	—
x_2	0	1	$\frac{5}{19}$	$\frac{-3}{19}$	$\frac{45}{19}$	
x_1	1	0	$\frac{-2}{19}$	$\frac{1}{3}$	$\frac{20}{19}$	

Z value

$0 - [-1 \times 0] = 0$

$-1 - [-1 \times 1] = -1 + 1 = 0$

$0 - [-1 \times \frac{5}{19}] = \frac{5}{19}$

$1 - [-1 \times -\frac{3}{19}] = 1 - \frac{3}{19} \Rightarrow \frac{19-3}{19} = \frac{16}{19}$

$10 - [-1 \times \frac{45}{19}] = 10 + \frac{45}{19} \Rightarrow \frac{190+45}{19}$

$\Rightarrow \frac{235}{19}$

$\frac{1}{5} + \frac{2}{15}$

$\frac{1 \times 3}{5 \times 3} + \frac{2}{15}$

$\frac{3}{15} + \frac{2}{15}$

$\frac{3+2}{15} = \frac{5}{15} = \frac{1}{3}$

x_1 value

$1 - [\frac{2}{5} \times 0] = 1$

$\frac{2}{5} - [\frac{2}{5} \times 1] = 0$

$0 - [\frac{2}{5} \times \frac{5}{19}] = -[\frac{10}{95}] = -\frac{2}{19}$

$\frac{1}{5} - [\frac{2}{5} \times \frac{-3}{19}] = \frac{-6}{45} = \ldots$

$= \frac{1}{3}$

$2 - [\frac{2}{5} \times \frac{45}{19}] = \frac{90}{95}\ \frac{18}{19}$

$= 2 - \frac{18}{19}$

$= \frac{38-18}{19} = \ldots$

Conclusion :-

The Iteration stop there is no entering variable into the solution.

Hence it is a "optimal table"

from the last iteration, the Value of $Z = \frac{235}{19}$

and $x_1 = \frac{20}{19}$ $x_2 = \frac{45}{19}$

$Z = 5x_1 + 3x_2$

$= 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) \Rightarrow \frac{100 + 135}{19}$

$\Rightarrow \frac{235}{19}$

problem :- 5

Max $z = 5x_1 - 4x_2 + 3x_2$ S/to $2x_1 + x_2 - 6x_3 = 20$,

$6x_1 + 5x_2 + 10x_3 \leq 76$, $8x_1 - 3x_2 + 6x_3 \leq 50$

By Big-m method

By TAHA method

check wheather the RHS value of the Constraints are positive

Consider the Constraints

they Can be Converted into equation

Consider the first Constraint,

it Can be written as

$$2x_1 + x_2 - 6x_3 + A_1 = 20$$

Consider the Second Constraint,

it Can be written as

$$6x_1 + 5x_2 + 10x_3 + S_2 = 76$$

Consider the third Constraint,

it Can be written as

$$8x_1 - 3x_2 + 6x_3 + S_3 = 50$$

Initialization :

put $x_1 = x_2 = x_3 = 0$ then

$A_1 = 20$, $S_2 = 76$, $S_3 = 50$

Consider the objective function it Can be written as

$$z = 5x_1 - 4x_2 + 3x_3$$

$z = 5x_1 - 4x_2 + 3x_3$

$= 5x_1 - 4x_2 + 3x_3 + (0)S_2 + (0)S_3 - mA_1$

$= 5x_1 - 4x_2 + 3x_3 - m(20 - 2x_1 - x_2 + 6x_3)$

$= 5x_1 - 4x_2 + 3x_3 - 20m - 2mx_1 + mx_2 - 6mx_3$

$= x_1[2m+5] + x_2(m-4) - x_3(6m-3) - 20m$

$z - x_1(2m+5) - x_2(m-4) + x_3(6m-3) = -20m$

Table :- 1

Basic variables	x_1	x_2	x_3	S_2	S_3	A_1	RHS value	Ratio
Z	$-(2m+5)$	$-(m-4)$	$(6m-3)$	0	0	0	$-20m$	—
S_2	6	5	10	1	0	0	76	$\frac{38}{3}$
S_3	8	-3	6	0	1	0	50	$\frac{25}{4}$
A_1	2	1	-6	0	0	1	20	10

Table :- 2

Basic variable	x_1	x_2	x_3	S_2	S_3	A_1	RHS value	Ratio
Z	0						$\frac{125}{4}$	
S_2	0	$29/4$	$\frac{1}{2}$	1	$-6/8$	0	$76/2$	
→ x_1	~~8~~ 1	$-\frac{3}{8}$	$\cancel{\frac{6}{8}}\ \frac{3}{4}$	0	$\frac{1}{8}$	0	$\frac{50}{8}\ \frac{25}{4}$	
A_1	0	$7/4$	$-\frac{15}{2}$	0	$-\frac{1}{4}$	1	$15/2$	

$-(2m+5)$ ... $+40$ ... $+5$

Z value :-

$-(2m+5) - [-(2m+5) \times 1] = -(2m+5) + (2m+5) = 0$

$-(m-4) - [-(2m+5) \times -\frac{3}{8}] =$

$-(6m-3) - [-(2m+5) \times \frac{3}{4}] =$

$0 - [-(2m+5) \times 0] = 0$

$0 - [-(2m+5) \times \frac{1}{8}] = \frac{2m+5}{8}$

$0 - [-(2m+5) \times 0] = 0$

$-20m - [-(2m+5) \times \frac{25}{4}] =$

(2) $= (m-4) - \left(\frac{6m+15}{8}\right)$

(3) $-(6m-3) + \frac{6m+15}{4} \Rightarrow \frac{-12+15}{4} \Rightarrow \frac{3}{4}$

$25 \times 5 = 125$

(5) $\Rightarrow \frac{2m+5}{8}$

$-20m - \left[-\frac{50m+125}{4}\right]$

$\frac{-80m+50m+125}{4} \Rightarrow \frac{-30m+125}{4}$

S_2 :-

$6 - [6 \times 1) = 0$

$5 - [6 \times -3/8) = 5 - (-\frac{18}{8}) = \frac{40+18}{8} = \frac{58}{8} = \frac{29}{4}$

$10 - [6 \times \frac{3}{4}) = 10 - (\frac{18}{4}) = \frac{40-18}{4} = \frac{22}{4} = \frac{11}{2}$

$1 - [6 \times 0) = 1$

$0 - [6 \times \frac{1}{8}) = -\frac{6}{8}$

$0 - [6 \times 0) = 0$

$76 - [6 \times \frac{25}{4}] = \frac{76+150}{4} = \frac{304-150}{4} = \frac{154}{4} = \frac{77}{2}$

$25 \times 6 = 150$

$76 \times 4 = 304$

$304 - 150 = 154$

$2 - [2 \times 1) = 0$

$1 - [2 \times -\frac{3}{8}) = 1 + \frac{6}{8} = \frac{8+6}{8} = \frac{14}{8} = \frac{7}{4}$

$-6 - [2 \times 3/4) = -6 - \frac{6}{4} = \frac{-24-6}{4} = \frac{-30}{4} = \frac{-15}{2}$

$0 - (2 \times 0) = 0$

$0 - (2 \times 1/8) = -\frac{2}{8} = -\frac{1}{4}$

$1 - (2 \times 0) = 1$

$20 - (2 \times \frac{25}{4}) = 20 - \frac{50}{4} = \frac{80-50}{4} = \frac{30}{4} = \frac{15}{2}$

① Formulation of LPP :-

for Maximization

A Company manufacture three items these items are processed on three machines m_1, m_2 and m_3. The time require for each product on each machine is given below. Also the total time availability for each machine is also give. the company gets profit of rupees 20, 30 and 40 on every unit of the product A, B and C find the number of units of each product to be produced per a day such that the total ~~product~~ profit is maximum

Machine	Time per production			Avalibilty of time for machine
	A	B	C	
M_1	1	1.5	2	18
M_2	1	1	1	20
M_3.	1	2	2	16

sol :-

The purpose of objective manufacturing industry to maximaize the production [Maximize the profit]. Let

x_1 be the Number of units of product A

x_2 be the Number of units of product B

x_3 be the Number of units of product C

The profit per product are

Rs 20, Rs 30 & Rs 40 on each unit

the objective function is

$$\text{Max } z = 20x_1 + 30x_2 + 40x_3$$

S/to Constraints [availability of machine hours for production)

$$1x_1 + 1.5x_2 + 2x_3 \leq 18$$

$$1x_1 + 1x_2 + 1x_3 \leq 20$$

$$1x_1 + 2x_2 + 2x_3 \leq 16$$

and non - negativity Constraints

$$x_1 \geq 0,\ x_2 \geq 0 \ \& \ x_3 \geq 0$$

Problem (2)

food type	yield perunit			Cost per unit of food
	Protein	fat	Carbo Hyderate	
Bread	4	1	2	12
Butter	3	2	1	60
milk	3	2	4	17
Minimum require	5	2	3	

(2) formulation of LPP for Minimization :-

In our daily time proteins, fat and Carbo Hydrate are require with a minimum of 5, 2, 3 units respectively we take a food stuff Consisting of Bread, butter and milk the ills of nutrition requirement & Cost per unit of food items is given below it require to find the Combination of food items such that minimum requirement is satisfied and the total Cost of food is the minimum.

Sol the purpose of diet plan is to minimize to food Stuff will taken in the qunatity

Let $x_1 \rightarrow$ (represents) the mimimum quantity of bread to be Consmu

Let $x_2 \rightarrow$ the mimimum quantity of butter to be Consumed

$x_3 \rightarrow$ The minimum quantity of milk to be Consumed

The objective function as follows

$$\text{Min } z = 12x_1 + 60x_2 + 17x_3$$

and non negative Constraints

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

Design of Constraint set :-

S/to $4x_1 + 3x_2 + 3x_3 \geq 5$

$x_1 + 2x_2 + 2x_3 \geq 2$

$2x_1 + x_2 + 4x_3 \geq 3$

The standraded form Lpp as follows

$\text{Min } z = 12x_1 + 60x_2 + 17x_3$ and

S/to $4x_1 + 3x_2 + 3x_2 \geq 5$

$x_1 + 2x_2 + 2x_3 \geq 2$

$2x_1 + x_2 + 4x_3 \geq 3$

and Non negative Constrain $x_1 \geq 0$, $x_2 \geq 0$ & $x_3 \geq 0$

Advaned Topics in Lpp :-

Duality Concept :-

In a Lpp involved large number of variables and Constraints it Can be Solved by Known methods. The senstivity analysis of problem is to study the effect of Changes in various resource levels. Every Lpp has an Associated with another form of Lpp the original and given Problem is known as "primal problem". The Another form is known as "Dual problem". The solution to one problem will provide the solution to another problem indirectly. In few Cases the solution is simple easy and less Compltion time for dual problem lather than primal problem.

Primal problem.	Dual problem
a) Min $Z = C^T \cdot X$	Max $W = b^T \cdot Y$
S/to $A \cdot X \geq b$ $\Rightarrow$	S/to $A^T \cdot Y \leq C$
and $X \geq 0$	and $Y \geq 0$
b) Max $Z = C^T \cdot X$	Min $W = b^T \cdot Y$
S/to $A \cdot X \leq b$	S/to $A^T \cdot Y \geq C$
and $X \geq 0$	and $Y \geq 0$

Hw

(3) ABC food Company is developing a food suppliment Known as HI-PRO the Specification and Calories, proteins, vitamin Consists of three basic foods as given below find the quantities of f_1, f_2, f_3 should be combined such that the minimum diet requerement and minimum Cost will be achieived

Nutritional Elements	units nutritional elements Per 100gm basic food			Hi-pro Specifications
	f_1	f_2	f_3	
Calories	350	250	200	300
proteins	250	300	150	200
Vitamin A	100	150	75	100
Vitamin B	75	125	150	100
Cost per 100grms (Rs)	1.50	2.00	1.20	

The purpose of ABC food Company is to mimize

Let $x_1 \rightarrow$ the minimum quantity of calories

$x_2 \rightarrow$ The quantity of food f_2 to be used

$x_3 \rightarrow$

$x_4 \rightarrow$

objective function as follows

$\quad\quad 1.5 \quad\quad 1.5x_1 + 2.00x_2 + 1.20x_3$

$Min\ z = \cancel{300}x_1 \cancel{+ 200x_2 +} 100x_3 \cancel{+ 100x_4}$

S/to and non-negative constraints

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$

Design of constraint set :-

S/to $350x_1 + 250x_2 + 200x_3 \geq 300$

$250x_1 + 300x_2 + 150x_3 \geq 200$

$100x_1 + 150x_2 + 75x_3 \geq 100$

$75x_1 + 125x_2 + 150x_3 \geq 100$

Duality Concept :-

Problem :- 1

write Dual form of the following

$Min\ Z_x = 5x_1 + 2x_2 + x_3$

S/to $2x_1 + 3x_2 + x_3 \geq 20$

$6x_1 + 8x_2 + 5x_2 \geq 80$

$7x_1 + x_2 + 3x_3 \geq 40$

$x_1 + 2x_2 + 4x_3 \geq 50$

and $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$

Sol) The given problem is in a standrad form (Symmetric form) Since there are four Constraints in the primal there will be four variables in the DUAL problem.

Let this variables will be y_1, y_2, y_3 & y_4 respectively the dual form of the given problem is as below

$$\text{Max } Z_y = 50y_1 + 40y_2 + 30y_3 + 20y_4$$

S|to

$$1y_1 + 7y_2 + 6y_3 + 2y_4 \le 5$$

$$2y_1 + y_2 + 8y_3 + 3y_4 \le 2$$

$$4y_1 + 3y_2 + 5y_3 + y_4 \le 1$$

and $y_1 \ge 0$, $y_2 \ge 0$, $y_3 \ge 0$, $y_4 \ge 0$

Problem 2 :-

write Dual form $\text{Max } Z_x = 2x_1 + 5x_2 + 3x_2$

S|to $2x_1 + 4x_2 - 3x_3 \le 8$, $-2x_1 - 2x_2 + 3x_3 \ge -7$

$x_1 + 3x_2 - 5x_3 \ge -2$, $4x_1 + x_2 + 3x_3 \le 4$

and $x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$

Sol :- The Given (primal) LPP is not in symmetric form Convert it into standard form as below

$$\text{Max } Z_x = 2x_1 + 5x_2 + 3x_3$$

S|to

$$2x_1 + 4x_2 - 3x_3 \le 8$$

$$-2x_1 - 2x_2 + 3x_3 \ge 7 \Rightarrow 2x_1 + 2x_2 - 3x_3 \le 7$$

$$-x_1 - 3x_2 + 5x_3 \le 2 \text{ (minus removed)}$$

$$4x_1 + x_2 + 3x_3 \le 4$$

and $x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$

The DUAL FORM of given PRIMAL as below

$$\text{Min } Z_y = 8y_1 + 7y_2 + 2y_3 + 4y_4$$

S|to

$$2y_1 + 2y_2 - y_3 + 4y_4 \ge 2$$

$$4y_1 + 2y_2 - 3y_3 + y_4 \ge 5$$

$$-3y_1 - 3y_2 + 5y_3 + 3y_4 \ge 3$$

and $y_1 \ge 0$, $y_2 \ge 0$, $y_3 \ge 0$

Problem 3 :-

Max $z = 4x_1 + 2x_2$, S|to Constraint $x_1 - 2x_2 \geq 2$, $x_1 + 2x_2 = 8$

$x_1 - x_2 \leq 10$ & $x_1 \geq 0$ $x_2 \geq 0$

Sol The give primal LPP is not in the Symmetric form

It Can be Converted as below

Consider the first Constraint , it Can be written as

$$x_1 - 2x_2 \geq 2$$

$$\Rightarrow -x_1 + 2x_2 \leq -2$$

Consider the Second Contraint , it Can be written as

$$x_1 + 2x_2 = 8$$

$$\Rightarrow x_1 + 2x_2 \leq 8 \Rightarrow x_1 + 2x_2 \leq 8$$

$$x_1 + 2x_2 \geq 8 \Rightarrow -x_1 - 2x_2 \leq -8$$

The Symmetric of Standard form of PRIMAL LPP is a below

$$\text{Max } Z_x = 4x_1 + 2x_2$$

S|to
$$-x_1 + 2x_2 \leq -2$$
$$x_1 + 2x_2 \leq 8$$
$$-x_1 - 2x_2 \leq -8$$
$$x_1 - x_2 \leq 10$$

The dual form of given primal as follows

$$\text{Min } Z_Y = 10Y_1 - 8Y_2 + 8Y_3 - 2Y_4$$

S|to
$$Y_1 - Y_2 + Y_3 - Y_4 \geq 4$$
$$-Y_1 - 2Y_2 + 2Y_3 + 2Y_4 \geq 2$$

and $Y_1 \geq 0$, $Y_2 \geq 0$, $Y_3 \geq 0$, $Y_4 \geq 0$

put $Y = (-Y_2 + Y_3)$

$$\Rightarrow Y_1 + Y - Y_4 \geq 4$$
$$-Y_1 + 2Y + 2Y_4 \geq 2$$

$-Y_2 + Y_3$

$-(Y_2 - Y_3)$

and $y_1 \geq 0, y_4 \geq 0$

and y is UN-RESTRICTED (It may +ve -ve '0')

The objective function can be written as

$$\text{Min } z_y = 10y_1 + 8y - 2y_4$$

$$y_1 + y - y_4 \geq 4$$

$$-y_1 + 2y + 2y_4 \geq 2$$

and $y_1 \geq 0, \; y_4 \geq 0$

y is UN-RESTRICTED

problem 4 :-

$$\text{Max}_z = 5x_1 + 12x_2 + 4x_3$$

S/to $\quad x_1 + 2x_2 + x_3 \leq 10$

$$2x_1 - x_2 + 3x_3 = 8$$

and $\quad x_1, x_2, x_3 \geq 0$

The given primal LPP is not in the symmetric form

It can be converted as below

Consider the first constraint

$$x_1 + 2x_2 + x_3 \leq 10$$

Consider the second constraint it can be written as

$$2x_1 - x_2 + 3x_3 = 8$$

$$\Rightarrow 2x_1 - x_2 + 3x_3 \leq 8 \Rightarrow 2x_1 - x_2 + 3x_3 \leq 8$$

$$2x_1 - x_2 + 3x_3 \geq 8 \Rightarrow -2x_1 + x_2 - 3x_3 \leq -8$$

~~Consider the third constraint it can be written as~~

~~the dual form of given primal as follows~~

The symmetric of standard form of primal LPP is below

$$\text{Max } z_x = 5x_1 + 12x_2 + 4x_3$$

S/to $\quad x_1 + 2x_2 + x_3 \leq 10$

$$2x_1 - x_2 + 3x_3 \leq 8$$

$$-2x_1 + x_2 - 3x_3 \leq -8$$

The dual form of given primal as follows

$$\text{Min } z_y = -8y_1 + 8y_2 + 10y_3$$

S/to $-2y_1 + 2y_2 + y_3 \geq 5$

$y_1 - y_2 + 2y_3 \geq 12$

$-3y_1 + 3y_2 + y_3 \geq 4$

min≥
max≤

and $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$

problem 7 :-

min $z = 2x_1 + 3x_2 + 4x_3$ S/to $2x_1 + 3x_2 + 5x_3 \geq 2$, $8x_1 + x_2 + 7x_3 = 3$, $x_1 + 4x_2 + 6x_3 \leq 5$ and $x_1 \geq 0$, $x_2 \geq 0$

the given primal problem is un restrcted variable it Can be written as. let $x_3 = x_3' - x_3''$

where $x_3' \geq 0$ and $x_3'' \geq 0$

The objective function Can be written as sub x_3 value

$$\text{Min } z_x = 2x_1 + 3x_2 + 4(x'_3 - x''_3)$$

$$= 2x_1 + 3x_2 + 4x''_3$$

Consider the first constraint i.e $2x_1 + 3x_2 + 5x_3 \geq 2$

Consider the second Constraint it Can be written as

$\Rightarrow 3x_1 + x_2 + 7x''_3 \geq 3$

$\Rightarrow 3x_1 + x_2 + 7x_3 \leq 3 \Rightarrow -3x_1 - x_2 - 7x''_3 \geq -3$

Consider the third Constraint it Can be written as

$x_1 + 4x_2 + 6x_3 \leq 5$

$\Rightarrow -x_1 - 4x_2 - 6x''_3 \geq -5$

The Symmetric standrad form of primal LPP is

$$\text{Min } z_x = 2x_1 + 3x_2 + 4x''_3$$

S/to $2x_1 + 3x_2 + 5x''_3 \geq 2$

$3x_1 + x_2 + 7x''_3 \geq 3$

$-3x_1 - x_2 - 7x''_3 \geq -3$

$-x_1 - 4x_2 - 6x''_3 \geq -5$

Problem

5) $\max z = 2x_1 + 3x_2 + x_3$

S/to $4x_1 + 3x_2 + x_3 = 6$, $x_1 + 2x_2 + 5x_3 = 4$ & $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

The given primal problem is not in the symmetric form

It can be converted as below.

Consider the first constraint

$$4x_1 + 3x_2 + x_3 = 6$$

$$\Rightarrow 4x_1 + 3x_2 + x_3 \leq 6$$

$$\Rightarrow 4x_1 + 3x_2 + x_3 \geq 6 \Rightarrow -4x_1 - 3x_2 - x_3 \leq -6$$

Consider the second constraint

$$x_1 + 2x_2 + 5x_3 = 4$$

$$\Rightarrow x_1 + 2x_2 + 5x_3 \leq 4$$

$$\Rightarrow x_1 + 2x_2 + 5x_3 \geq 4 \Rightarrow -x_1 - 2x_2 - 5x_3 \leq -4$$

The symmetric standrad form of primal LPP is

$$\max z_x = 2x_1 + 3x_2 + x_3$$

S/to

$$4x_1 + 3x_2 + x_3 \leq 6$$

$$-4x_1 - 3x_2 - x_3 \leq -6$$

$$x_1 + 2x_2 + 5x_3 \leq 4$$

$$-x_1 - 2x_2 - 5x_3 \leq -4$$

The dual form of given primal as follows

$$\text{Min } Z_Y = -6Y_1 + 4Y_2 - 8Y_3 + 6Y_4$$

S|to $-Y_1 + Y_2 - 4Y_3 + 4Y_4 \geq 2$

$-2Y_1 + 2Y_2 - 3Y_3 + 3Y_4 \geq 3$

$-5Y_1 + 5Y_2 - Y_3 + Y_4 \geq 1$ and $Y_1 \geq 0, Y_2 \geq 0, Y_3 \geq 0$

$Y_4 \geq 0$

Problem :-

6. $\text{Min } Z = 3x_1 - 2x_2 + 4x_3$

S|to $3x_1 + 5x_2 + 4x_3 \geq 7$, $6x_1 + x_3 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$, $x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$ and $x_1 \geq 0$, $x_2 \geq 0$ & $x_3 \geq 0$

The given primal problem is not in the symmetric form
It Can Converted as follow.

Consider the first Constraint

$\Rightarrow 3x_1 + 5x_2 + 4x_3 \geq 7$

Consider the second Constraint

$\Rightarrow 6x_1 + x_3 + 3x_3 \geq 4$

Consider the third Constraint it Can be written as

$7x_1 - 2x_2 - x_3 \leq 10$

$\Rightarrow -7x_1 + 2x_2 + x_3 \geq -10$

Consider the fourth Constraint

$\Rightarrow x_1 - 2x_2 + 5x_3 \geq 3$

Consider the fifth Constraint [it Can be written as]

$\Rightarrow 4x_1 + 7x_2 - 2x_3 \geq 2$

The symmetric standrad form of primal LPP is

$$\text{Min } Z_x = 3x_1 - 2x_2 + 4x_3$$

S|to $3x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_3 + 3x_3 \geq 4$

$-7x_1 + 2x_2 + x_3 \geq -10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$

The dual form of given primal as follows.

$$\max Z_y = 2y_1 + 3y_2 - 10y_3 + 4y_4 + 7y_5$$

S/to $4y_1 + y_2 - 7y_3 + 6y_4 + 3y_5 \le 3$

$7y_1 - 2y_2 + 2y_3 + y_4 + 5y_5 \le -2$

$-2y_1 + 5y_2 + y_3 + 3y_4 + 4y_5 \le 4$

and $y_1 \ge 0,\ y_2 \ge 0,\ y_3 \ge 0,\ y_4 \ge 0,\ y_5 \ge 0$