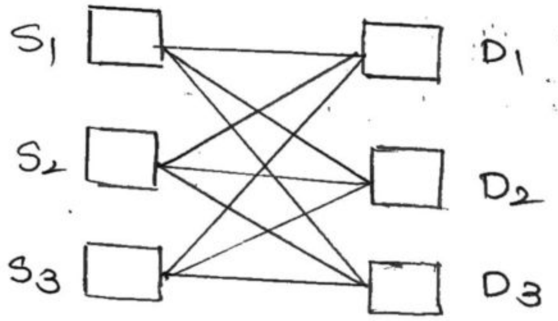
Unit - 2

Transportation Model



Source
Plant / Industry
S.K.U

Destination
ware House
show - Room
Retail - outlest Coustomer

The transportation model had processed the name because many of its application involve to determine optimally transport of materials.

Application of this algorithm requires a large number of Constraints and variables

Transportation Problem :

The model of transportation deals with distribution of a Commonity [produts / metrials] from a group of supply Centers known as "Sources", to another group of receiving Centers known as "destinations", In such way that to minimize the total Cost of transportation.

the problem is Represented in a matrix form the last Row will be "demand" for quantity to be received from the sources. The last Column will be the supply of quantity from sources to the destinations

The model is developed based on the following.

1. To get a balance b/w total supply from all sources and to total demand for all destinations.
2. The Cost of transportation of the quantity is directly propostional to the number of units (quantity) to be distributed,
3. For this model [Solution]

$\boxed{\text{Total supply} = \text{Total demand}}$

- The solution to the model will be in two phases
  - (i) Initial feasible Solution
  - (ii) Test for optimality

<u>Note</u> :-

The unit Cost of transportation is Show in the box, placed at left upper most Corner of the Cell

<u>Definition</u> :-

The transportation model is to transport various quantities of a single and homogeneous Commodity that are initially Store at various resources and to different destinations, In such a way that the total transportation Cost will be the minimum

Initial feasible <u>Solution</u> :-

<u>PHASE - I</u>

This phase Contains three methods, for finding Solution to the given transportation model (or) problem (or) algorithm they are

(a) North-west Corner method (NWCM)

(b) LEAST Cost method (LCM)

(c) Vogel's Approximation model (VAM)

PHASE - 2 :-

Test for optimality :-

In this phase the initial feasible Solution from the phase I is consider for testing the optimal Solution two methods are apply for finding the optimality they are.

a) stepping - stone method

b) modi method (modified distribution)

phase I :-

Initial feasible Solution

(a) North- west corner method (NWCM)

It is the simplest procedure to generate an inital feasible solution. It begins with North west (or) upper left Corner Cell of the given problem the procedure steps as follows :-

* step 1 :- Select the North west Corner cell of the given problem [matrix] and allocate as many units [quantity] as much quantity as possible [equal to the minimum b/w avaliable Supply and demand requirement]

* step 2 :- Adjust the Supply and demand numbers in the Corresponding row and Column allocations

* step 3 :- (a) If the supply for the first row is staisfy move down to the first Cell in the second row and first Column go to the step 2

(b) If the first column is satisfied then move horizontally to the next cell in the second column and first row go the step 2

* Step 4 :- for any cell if the supply equals to demand then next allocation can be done to the cell either in next row of column

* Step 5 :- Delete the row (or) column that is satisfy in the given matrix.

* Step 6 :- Continue the process until the total avaliable quantity is completely fully allocated to the require cell

problem :-

	D_1	D_2	D_3	Supply of quantity
S_1	7 (20)	6	9	20
S_2	5 (1)	7 (25)	3 (2)	~~28~~ ~~27~~ ~~2~~ 0
S_3	4	5	8 (17)	~~17~~ 0
Demand requirement	~~21~~ 1	~~25~~ 0	~~19~~ ~~17~~	65 / 65

row total Column total is equal

1+25+2

20 1

Sol:- Check whether

Total supply = Total Demand

The given model is " BAlACED " problem.

By North-west Corner method (NWCM)

The total cost of transportation (for occupied cells)

$\Rightarrow$ RS. $7 \times 20 + 5 \times 1 + 7 \times 25 + 3 \times 2 + 8 \times 17$

$\Rightarrow 140 + 5 + 175 + 6 + 136$

$\Rightarrow 462$

(b) Least Cost method :- [LCM]

The allocation is done Considering lowest unit Cost of the given problem. Hence these reduces processing time and work to find the feasible Solution.

The procedural steps are below..

Step 1 :- (a) select the cell with the lowest transportation Cost among all rows or Columns in the given problem

(b) If the minimum Cost is not unique select randomly or literally

Step 2 :- Allocate as many units as possible to the result in Cell from above step

Delete row or Column that is satisified. ~~repeat the~~.

Step 3 :- Repeat the above procedure until the allocation was Completed.

problem :-

	D_1	D_2	D_3	Supply of Qty
S_1	7	6 (20)	9	~~20~~ 0
S_2	5 (4)	7 (5)	3 (19)	~~28~~ ~~9~~ 5
S_3	4 (17)	5	8	~~17~~ 0
Demand requirement.	~~21~~ 4	~~25~~ 5	~~19~~ 0	65 / 65

Sol :-

Check whether the Condition is Satisfied

Total supply = Total demand

By least Cost Method

the total Cost of transportation

(for occupied cells)

$\Rightarrow$ Rs $6 \times 20 + 5 \times 4 + 7 \times 5 + 3 \times 19 + 4 \times 17$

$\Rightarrow$ Rs 300

(c) Vogel's Approximation model (VAM) :-

This model is prefer because the initial feasible solution derived from this method is very close to the optimal Solution or it may be the optimal solution.

Hence the Computation process and time is redused in the phase 2 the procedural steps as follows.

Step 1 :- Calculate the penulty for every row and Column for the given problem.

Penulty is the difference b/w Smallest unit Cost and next Smallest unit Cost of the specif Selected row (or) column

Step 2 :- (a) Identify the row (or) Column with the largest penulty of the given problem. In that row (or) Column Select the smallest unit Cost Cell and allocate as much quantity as possible.

Delete the row (or) Column that is satisfy

(b) If the largest penulty not unit select randomly

Step 3 :- Repeat the above process until the allocation was Completed.

				Supply	Penalty
	7	6 (20)	9	~~20~~	1 1
	5 (9)	7	3 (19)	~~28~~ 9	2 2 (2)
	4 (12)	5 (5)	8	~~17~~ 5	1 1
Demand	~~21~~ ~~12~~ 0	25 20	~~19~~ 0	65 / 65	
Penalty	1	1	5 (1)		
	1	1			
	3 (3)				

Sol

Check whether the condition is satisfied

Total supply = Total demand

The total transportation cost

(for occupied cell)

$\Rightarrow$ Rs $6 \times 20 + 5 \times 9 + 3 \times 19 + 4 \times 12 + 5 \times 5$

$\Rightarrow$ Rs $120 + 45 + 57 + 48 + 25$

$\Rightarrow$ Rs 295

How

Find the problem.

	D1	D2	D3	D4	Supply
S1	19 (5)	30 (2)	50	10	~~7~~ ~~2~~ 0
S2	70	30 (6)	40 (3)	60	~~9~~ ~~3~~ 0
S3	40	8	70 (4)	20 (14)	~~18~~ 14
Demand	~~5~~ 0	~~8~~ ~~6~~ 0	~~7~~ ~~4~~ 0	~~14~~ 0	34 / 34

a) North west corner method :-

check wheather the Condition is satisfied

Total Quantity = Total Demand

By North west corner method

The total cost of transportation is

$\Rightarrow$ Rs $19 \times 5 + 30 \times 2 + 30 \times 6 + 40 \times 3 + 70 \times 4 + 20 \times 14$

$\Rightarrow$ Rs $95 + 60 + 180 + 120 + 280 + 280$

$\Rightarrow$ Rs 1015

b) Least cost method :-

	D1	D2	D3	D4	supply
S_1	19	30	50	10 (7)	~~7~~ 0
S_2	70 (2)	30	40 (7)	60	~~9~~ ~~7~~ 0
S_3	40 (3)	8 (8)	70	20 (7)	~~18~~ ~~10~~ 3
Demand	~~5~~ ~~7~~ 0	~~8~~ 0	~~7~~ 0	~~14~~ ~~7~~ 0	34 / 34

check wheather the Condition is satisfied

Total Quantity = Total demand

By least cost method

Total Cost of transportation is

$\Rightarrow$ Rs $10 \times 7 + 7 \times 2 + 40 \times 7 + 40 \times 3 + 8 \times 8 + 20 \times 70$

$\Rightarrow$ Rs $70 + 140 + 280 + 120 + 64 + 140$

$\Rightarrow$ Rs 814

c. Vogels approximate method :-

	D1	D2	D3	D4	Supply	penality				
S1	19 (5)	30	50	10 (2)	~~7~~ ~~2~~ 0	9	9	40	40	h
S2	70	30	40 (7)	60 (2)	~~9~~ ~~2~~ 0	10	20	20	20 ← (3)	
S3	40	8 (8)	70	20 (10)	~~18~~ ~~10~~ 0	12	20	50 ↑ (2)	X	x
Demand	~~5~~ 0	~~8~~ 0	~~7~~ 0	~~14~~ ~~4~~ ~~2~~ 0	34 / 34					
	21	22 ← (1)	10	10						
	21 ← (2)	x	10	10						
	x	x	10	10						
	x	x	10	50 ← (4)						

Check wheather the Condition is satisfied

Total quantity = Total demand

By volgels approximate method

Total cost of transportation is

$\Rightarrow$ Rs $19 \times 5 + 10 \times 2 + 40 \times 7 + 60 \times 2 + 8 \times 8 + 20 \times 10$

$\Rightarrow$ Rs 779

Phase - 2 :- [Test for optimality]

The following two techinques are apply to find the optimal solution

a. stepping - stone method

b. modi method

b. Modi Method :- 'U, V' method

* the procedural steps as follows

step 1 :- Check or inspect whether the number of occupied Cells is greater than or equal to $(m+n-1)$ where, m = number of rows and n = number of columns in the given matrix

step 2 :- If the Condition is not satisfy the problem suffers with "Degeneracy".

Step 3 :- Remove the degeneracy at initial feasible solution stage

Step 4 :- Find the value to the following relation to the, occupied cells as - "($c_{ij} = u_i + v_j$)" where

c_{ij} is the unit transportation cost of that occupied cells.

u_i = Cell number of i^{th} Row and v_j = Cell number of j^{th} Column

Step 5 :- Find all values of u_i and v_j [for occupied cells]

Step 6 :- Find the value of the relation net cost change to relation as "($c_i - u_i - v_j$)" for un-occupied cells

Step 7 :- If the net cost change values are either zero (or) positive, optimual solution is derived

Step 8 :- Other wise the degeneracy is present in the phase 2. Remove the degeneracy and find the Solution to the given Problem.

Degeneracy :- [Defectiveness] error

The defectiveness occurs both the stages of the Solution method.

Defectiveness :-

a) At phase-1 (Initial feasible solution)

If the number of occupied cells is less than the relation $m+n-1$ the degeneracy is present. where 'm' number of rows. & 'n' number of columns

(b) At phase - 2 (optiamality Test) :-

If the net cost change value for un occupied cells is having negative value, then the degeneracy is present

Removal of degeneracy :-

(a) At phase - 1 :-

To remove the degenarcy in phase I an artifical quantity denoted by "Greek Letter" (ϵ)

where $\epsilon = 0.0000001$

To an unoccupied cell that has minimum unit cost of transportation.

(b) At phase - 2 :-

~~To~~ To Remove the degenarcy in the phase - 2 "Trace a closed loop path" and apply step in stone method and may it has an occupied cell.

Repeat the above procedure until all net cost changed values are zero (or) positive.

Problem :- 1

				Supply	
	7	6 (20)	9	20	1 1 1
	5 (9)	7	3 (19)	28	2 2 (2)
	4 (12)	8 (5)	8	17	1 1 1
Demand	21	25	19	65 / 65	
	(3) 1 3	1 1	5 ← (1)		

Check whether the Condition is satisfied

Total supply = Total demand

Given problem is a "balanced model"

phase - I :- Apply Vogel's approximate model

The total number of occupied cells is '5'

The value of the mathematical relation $(m+n-1)$

$$= 3+3-1 = 5$$

Since the number of occupied cells is = value of relation $(m+n-1)$.

- There is "no degeneracy".

Phase II :- (Test for optimality)

By modi method

	V1	V2	V3	supply
U1	7 11	6 20 12	9 13	20
U2	5 9 21	7 22	3 19 23	28
U3	4 12 31	5 5 32	8 33	17
Demand	21	25	19	65 / 65

for occupied ~~cell~~ find the value of relation.

$\Rightarrow C_{ij} = U_i + V_j$

$C_{12} = U_1 + V_2 = 6$

$C_{21} = U_2 + V_1 = 5$

$C_{23} = U_2 + V_3 = 3$

$C_{31} = U_3 + V_1 = 4$

$C_{32} = U_3 + V_2 = 5$

put $U_2 = 0$

$U_1 = 0$	$V_1 = 5$
$U_2 = 0$	$V_2 = 6$
$U_3 = -1$	$V_3 = 3$

The value of Net cost change (for un occupied cells)

$\Rightarrow c_{ij} - U_i - V_j$

$C_{11} - U_1 - V_1 = 7 - 0 - 5$

$= 2$ (+ve)

$C_{13} - U_1 - V_3 = 9 - 0 - 3 = 6$ (+ve)

$C_{22} - U_2 - V_2 = 7 - 0 - 6 = 1$ (+ve)

$C_{33} - U_3 - V_3 = 8 - (-1) - 3 = 6$ (+ve)

Conclusion :-

Since, All values of net Cost change relation [for unoccupied cell] are positive optimal solution is reached or uptained Hence, " It is the optimal allocation"

The total transportation Cost = rupess $6 \times 20 + 5 \times 9 + 3 \times 19 + 4 \times 12 + 5 \times 5$

= rs 295

problem :-

solve the following.

2	7	4	5
3	3	1	8
5	4	7	7
1	6	2	14
7	9	18	

check the following condition

Total supply = total demand

Hence the given problem is a "BAIANCED MODEI"

phase I :- [Initial feasible solution stage]

Calculate by VAM method

				supply	penality			
	2 (3)	7 (2)	4	~~5~~ 2	2	2	5	5
	3	3	1 (8)	~~8~~	~~2~~ ← ①		×	
	5	4 (7)	7	~~7~~ 0	1	1	1	1
	1 (4)	6	2 (10)	~~14~~ ~~4~~ 0	1	1	5 ↑ ③	
Demand	~~7~~ ~~3~~ 0	~~9~~ 2	~~18~~ ~~10~~ 0	34				
	34							
	1	1	1					
	1	1	2 ← ②					
	3	3						

The number of occupied cells = 6

the value to the relation $(m+n-1) \Rightarrow 4+3-1$

where m is no of rows $\Rightarrow 7-1$

n is no of columns $\Rightarrow 6$

Since the number of occupied cells is EQUAl TO to value of the relation $m+n-1$

There is " NO DEGENERACY "

phase 2 :- (Test for optimality)

By Modi method

	V_1	V_2	V_3	
U_1	2 (3) 11	7 (2) 12	4 13	5
U_2	3 21	3 22	1 (8) 23	8
U_3	5 31	4 (7) 32	7 33	7
U_4	1 (4) 41	6 42	2 (10) 43	14
	7	9	18	34 / 34

$2 \times 3 + 7 \times 2 + 8 + 4 \times 7 + 4 + 2 \times 10$

$6 + 14 + 8 + 28 + 4 + 20$

$$\begin{array}{r} 48 \\ 20 \\ 12 \\ \hline 80 \end{array}$$

for occupied cells

The value of the mathematical relation

$C_{ij} = U_i + V_j$

$C_{11} = U_1 + V_1 = 2$

$C_{12} = U_1 + V_2 = 7$

$C_{23} = U_2 + V_3 = 1$

$C_{32} = U_3 + V_2 = 4$

$C_{41} = U_4 + V_1 = 1$

$C_{43} = U_4 + V_3 = 2$

put $U_1 = 0$

$U_1 = 0$ | $V_1 = 2$

$U_2 = -2$ | $V_2 = 7$

$U_3 = -3$ | $V_3 = 3$

$U_4 = -1$

$V_3 + 7 = 4$

$V_3 = 4 - 7$

$U_3 = -3$

for unoccupied cells :-

Find the value of Net cost change

$C_{ij} - U_i - V_j$

$C_{13} - U_1 - V_3 = 4 - 0 - 3 = 1$ (+ve)

$C_{21} - U_2 - V_1 = 3 + 2 - 2 = 3$ (+ve)

$C_{22} - U_2 - V_2 = 3 + 2 - 7 = 5 - 7 = -2$ (-ve)

$C_{31} - U_3 - V_1 = 5 + 3 - 2 = 5 + 1 = 6$ (+ve)

$C_{33} - U_3 - V_3 = 7 + 3 - 3 = 7$ (+ve)

$C_{42} - U_4 - V_2 = 6 + 1 - 7 = 0$

od -ve pole 2 4 8

2+3   2-2

2 (5)	7 (0)	4	5
3	3 (2)	1 (6)	8-2 8
5	4 (7)	7	7
1 (2)	6	2 (12)	14
4-2 7	9	10+2 18	

unoccupied cell means [illegible] is zero

occupied cell [illegible]

2 4 8
select least value
so we take 2

phase - I

(Initial feasible solution)

The number of occupied cell = 6

The value to the relation $(m+n-1) \Rightarrow 4+3-1$
$\Rightarrow 6$

Since the number of occupied cell = value of relation $m+n-1$

There is No "DEGENERACY"

Phase II :- (Test for optimality)

for occupied cells

the value of the mathematical relation

$C_{ij} = U_i + V_j$

$C_{11} = U_1 + V_1 = 2$

$C_{22} = U_2 + V_2 = 3$

$C_{23} = U_2 + V_3 = 1$

$C_{32} = U_3 + V_2 = 4$

$C_{41} = U_4 + V_1 = 1$

$C_{43} = U_4 + V_3 = 2$

let $U_2 = 0$

$U_1 = 0 \quad V_1 = 2$

$U_2 = 0 \quad V_2 = 3$

$U_3 = 1 \quad V_3 = 1$

$U_4 = -1$

for un-occupied cell

The Net cost change value are

$(C_{ij} - U_i - V_j)$

$C_{12} - U_1 - V_2 = 7 - 0 - 3 = 4$ (+ve)

$C_{13} - U_1 - V_3 = 4 - 0 - 1 = 3$ (+ve)

$C_{21} - U_2 - V_1 = 3 - 0 - 2 = 1$ (+ve)

$C_{31} - U_3 - V_1 = 5 - 1 - 2 = 2$ (+ve)

$C_{33} - U_3 - V_3 = 7 - 1 - 1 = 5$ (+ve)

$C_{42} - U_4 - V_2 = 6 - (-1) - 3 = 4$ (+ve)

Since all net cost change values are +ve for un occupied cells

the optimal solution is obtain

30
30
16
76

HENCE IT IS THE OPTIMAL ALLOCATION

The total transportation cost for occupied cell

$\Rightarrow 2 \times 5 + 3 \times 2 + 1 \times 6 + 4 \times 7 + 1 \times 2 + 2 \times 12$

$\Rightarrow 10 + 6 + 6 + 28 + 2 + 24 \Rightarrow 76$

Problem :-

				Supply
11	13	17	14	250
16	18	14	10	300
21	24	13	10	400
Demand 200	225	275	250	950 / 950

Check the following condition

Total supply = total demand

Hence the given problem is a " Balanced model "

Phase I :- [Initial feasible solution]

Calculate by VAM method.

				supply	penality
11 (200)	13 (50)	17	14	250	~~2~~ 3 ← x
16	18 (175)	14	10 (125)	300	4 4 4 ← (2)
21	24	13 (275)	10 (125)	400	3 3 3 ← (4)
(200) 175	225	275	~~250~~ 125		
(1) → 5	5	1	4		
	5 (3)	1	4		
		1	4		
		(5)			

The number of occuiped Cells = 6

The value to the relation $(m+n-1) = 3+4-1$

m is no of rows $= 7-1$

n is no of Columns $= 6$

Since the number of occupied cells is equal to value of the relation $(m+n-1)$

Phase -2 :- (Test for optimality)

By Modi method

for occupied cells

The value of the relation $C_{ij} = U_i + V_j$

$C_{11} = U_1 + V_1 = 11$	put $u_1 = 0$
$C_{12} = U_1 + V_2 = 13$	$u_1 = 0$ $V_1 = 11$
$C_{22} = U_2 + V_2 = 18$	$U_2 = 5$ $V_2 = 13$
$C_{24} = U_2 + V_4 = 10$	$U_3 = 5$ $V_3 = 8$
$C_{33} = U_3 + V_3 = 13$	$V_4 = 5$
$C_{34} = U_3 + V_4 = 10$	

The value of Net cost change for un-occupied cells

$C_{ij} - U_i - V_j$

$C_{13} - U_1 - V_3 = 17 - 0 - 8 = 9$ (+ve)

$C_{14} - U_1 - V_4 = 14 - 0 - 5 = 9$ (+ve)

$C_{21} - U_2 - V_1 = 16 - 5 - 11 = 16 - 16 = 0$

$C_{23} - U_2 - V_3 = 14 - 5 - 8 = 14 - 13 = 1$ (+ve)

$C_{31} - U_3 - V_1 = 21 - 5 - 11 = 21 - 16 = 5$ (+ve)

$C_{32} - U_3 - V_2 = 24 - 5 - 13 = 24 - 18 = 6$ (+ve)

Conclusion :- Since, all the values of net cost change relation are positive for "un occupied" Cells the optimal solution is obtain Hence it is the "Optimal Allocation"

The total transportation cost for occupied Cells is

Rs $11 \times 200 + 13 \times 50 + 18 \times 175 + 10 \times 125 + 13 \times 275 + 10 \times 125$

Rs $2200 + 650 + 3150 + 1250 + 3575 + 1250$

Rs 12075 /-

problem :—

Un-balanced models :—

				supply
	1	2	3	10
	4	5	6	11
	7	8	9	12
Demand	13	14	15	33 / 42

sol check the following Condition

Total supply = total demand

The given problem is not a balanced model

ADD A Row to Balance

The problem Can be modified an

1	2	3	10
4	5	6	11
7	8	9	12
0	0	0	9
13	14	15	42 / 42

The problem Can be modified an

1	2	3	0	13
4	5	6	0	14
7	8	9	0	15
10	11	12	9	42 / 42

A Computer manufacture has decided to lunch an adversting Campaign of television, magazine and radio. It is estimated the maximum exposeure for this media will be 70, 50 and 40 million respectivi according to a market servey, It was found that the minimum decide exposere with in age groups 15-20, 21-25, 26-30, 31-35 and above 35 or 10, 20, 25, 35, and 55 million respecitively. the table below givs the estimated Cost in Paise per explaser for each of the media Plan to minimize the Cost

media	Age groups				
	15-20	21-25	26-30	31-35	above 35
Tv	14	9	11	11	12
Radio	11	7	6	7	8
magazine	9	10	7	10	8

Solve the problem and find the optimal solution i.e maximum Coverage at minimum Cost ?

sol

14	9	11	11	12	70
11	7	6	7	8	50
9	10	7	10	8	40
10	20	25	35	55	145 \ 160

55

phase I :- [Initial feasible solution]

Calculate by VAM method

						Supply	Row penalties
14	9 (20)	11	11	12 (35)	0 (15)	~~70~~ ~~55~~ 35 0	9 2 2 2 1
11	7	6 (15)	7 (35)	8	0	~~50~~ 15 0	6 1 1 1 2
9 (10)	10	7 (10)	10	8 (20)	0	~~40~~ 30 20 0	7 1 1 1 1
10	20	~~25~~ 10	35	~~55~~ 35 0	15	160	

(1) (4) (5)

2	2	1	3	0	0
2	2	1	(2) → 3	0	X
(3) → 2	2	1	X	0	X
X	2	1	X	0	X
X	X	1	X	0	X

The no of occupied cells = 8

The value of the relation $(m+n-1) \Rightarrow 3+6-1 \Rightarrow 9-1 \Rightarrow 8$

Since no of occupied cells is equal to the relation $(m+n-1)$

There is NO- DEGENERACY"

phase II :- (Test for optimality)

for occupied cells, The value of the mathematical relation

$c_{ij} = u_i + v_j$

$C_{12} = u_1 + v_2 = 9$

$C_{15} = u_1 + v_5 = 12$

$C_{16} = u_1 + v_6 = 0$

$C_{23} = u_2 + v_3 = 6$

$C_{24} = u_2 + v_4 = 7$

$C_{31} = u_3 + v_1 = 9$

$C_{33} = u_3 + v_3 = 7$

$C_{35} = u_3 + v_5 = 8$

put $u_3 = 0$

$u_1 = 4$ | $v_1 = 9$

$u_2 = -1$ | $v_2 = 5$

$u_3 = 0$ | $v_3 = 7$

$v_4 = 5$

$v_5 = 8$

$v_6 = -4$

for unoccupied cells,

The net cost change value are

$C_{11} - u_1 - v_1 = 14 - 4 - 9$
$= 14 - 13 \Rightarrow 1 (+ve)$

$C_{13} - u_1 - v_3 = 11 - 4 - 7$
$= 11 - 11 \Rightarrow 0$

$C_{14} - u_1 - v_4 = 11 - 4 - 5$
$= 11 - 9 \Rightarrow 2 (+ve)$

$C_{21} - u_2 - v_1 = 11 + 1 - 9 \Rightarrow 3 (+ve)$

$C_{22} - u_2 - v_2 = 7 + 1 - 5 \Rightarrow 3 (+ve)$

$C_{25} - u_2 - v_5 = 8 + 1 - 8 \Rightarrow 1 (+ve)$

$C_{26} - u_2 - v_6 = 0 + 1 + 4 \Rightarrow 5 (+ve)$

$C_{32} - u_3 - v_2 = 10 - 0 - 5 \Rightarrow 5 (+ve)$

$C_{34} - u_3 - v_4 = 10 - 0 - 5 \Rightarrow 5 (+ve)$

$C_{36} - u_3 - v_6 = 0 - 0 + 4 \Rightarrow 4 (+ve)$

Conclusion :- since, all the values of net cost change relation are +ve for "un occupied" cell the optimal solution is obtain.

Hence, It is the "Optimall Allocation"

The total transportation cost for occupied cells

$\Rightarrow$ Rs $9\times20 + 12\times35 + 0\times15 + 6\times15 + 7\times35 + 9\times10 + 7\times10 + 8\times20$

$\Rightarrow$ Rs $180 + 420 + 0 + 90 + 245 + 90 + 70 + 160$

$\Rightarrow$ Rs 1255

Nov, 2020 - MBA

Solve the following

			Supply
10	8	5	80
4	7	3	70
Demand 50	75	25	150 / 150

Check the following condition

Total supply = total demand

Hence the given problem is "Balanced model"

phase I :-

By VAM method

				Supply	Penality	
	10	8 (75)	5 (5)	~~80~~ ~~75~~ 0	3	3
	4 (50)	7	3 (20)	~~70~~ 20	1	4 ← (2)
Demand	50	75	~~25~~ ~~5~~ 0	150 / 150		
	6	1	2			
	(1) ↑	1	2			

The number of occupied cells = 4

The value to the relation $(m+n-1) \Rightarrow 2+3-1$

m is no of rows $\Rightarrow 5-1$

n is no of columns. $\Rightarrow 4$

Since the number of occupied cells is Equal to value of the relation $m+n-1$

There is "NO DEGENERACY"

phase II (Test for optimality)

By Modi method

for occupied cells.

The value of the relation $C_{ij} = u_i + v_j$

$U_1 = 0$

$C_{12} = U_1 + V_2 = 8$

$C_{13} = U_1 + V_3 = 5$

$C_{21} = U_2 + V_1 = 4$

$C_{23} = U_2 + V_3 = 3$

$U_1 = 0$ $\quad V_1 = 6$

$U_2 = -2$ $\quad V_2 = 8$

$V_3 = 5$

for the unoccupied cells

The Net Cost change value

$C_{ij} - U_i - V_j$

$C_{11} - U_1 - V_1 \Rightarrow 10 - 0 - 6$

$\Rightarrow 4 \text{ (+ve)}$

$C_{22} - U_2 - V_2 \Rightarrow 7 + 2 - 8$

$\Rightarrow 9 - 8$

$\Rightarrow 1 \text{ (+ve)}$

Since all the net cost change values positive optimal solution is reached

Hence is the optimal allocation

The total transportation cost for occupied cell

$\Rightarrow 8 \times 75 + 5 \times 5 + 4 \times 50 + 3 \times 20$

$\Rightarrow 600 + 25 + 200 + 60$

$\Rightarrow 885$

Un-Balanced model :-

					supply
4	2	3	2	6	8
5	4	5	2	1	12
6	5	4	7	5	14
Demand 4	4	6	8	8	34
				29	

For maximization :-

In general the transportation problem is moved towords minimizing the Cost of transportation for a given problem, ~~from~~ ~~the~~ various Sources to the different ~~esti~~ distinations However in few Cases the Objective may be minimizing the profit or Total output.

TO slove such problems the unit cost of transportation is change and the problem will be Sloved for minimization

Select the highest unit Cost [unit Cost of transportation] In the given Problem (or) Subtract all other elements / values [unit Cost] from it then the problem is Converted into minimization

Problem :- solve the following for maximization :-

15	51	42	33	23
80	42	26	81	44
90	40	66	60	33
23	31	16	30	

The given problem the objective for maximization.

convert the given problem into minimization

Select the highest cost in the given matrix and subtract all other values from it then the problem becomes minimization.

75	39	48	57	23
10	48	64	9	44
0	50	24	30	33
23	31	16	30	100 / 100

check the following condition

total supply = total demand

The given problem is a balanced model

phase I [Initial feasible stage]

By VAM method

calculate by VAM method.

75	39 (23)	48	57	~~23~~ 0	9	18	36	36
10 (6)	48 (8)	64	9 (30)	~~44~~ ~~14~~ 8	1	1	38	38
0 (17)	50	24 (16)	30	~~33~~ ~~17~~ 0	24	20	50 ↑(3)	X
~~23~~ ~~6~~ 0	~~31~~ ~~8~~	~~16~~ 0	~~30~~ 0	100 / 100				

10	9	24 ↑(1)	21
10	9	X	21 ← (2)
10	9	X	X
65 ↑(4)	9	X	X
X	9	X	X

The number of occupied cell = 6

The value to the relation $(m+n-1) = 3+4-1$

m is no of rows $= 6$

n is no of columns · there is "NO - DEGENERACY"

The value of the mathematical relation

$c_{ij} = U_i + V_j$

$C_{12} = U_1 + V_2 = 39$ $\quad U_2 = 0$

$C_{21} = U_2 + V_1 = 10$ $\quad U_1 = -9 \quad V_1 = 10$

$C_{22} = U_2 + V_2 = 48$ $\quad U_2 = 0 \quad V_2 = 48$

$C_{24} = U_2 + V_4 = 89$ $\quad U_3 = -10 \quad V_3 = 34$

$C_{31} = U_3 + V_1 = 0$

$C_{33} = U_3 + V_3 = 24$ $\quad V_4 = 89$

$\begin{array}{r} \overset{3}{\cancel{4}}18 - \\ 39 \\ \hline 9 \end{array}$

for occupied cells the Net cost change value $c_{ij} - U_i - V_j$

$C_{11} - U_1 - V_1 = 75 + 9 - 10 \Rightarrow 84 - 10 \Rightarrow 74$ (+ve)

$C_{13} - U_1 - V_3 = 48 + 9 - 34 \Rightarrow 57 - 34 \Rightarrow 23$ (+ve)

$C_{14} - U_1 - V_4 = 57 + 9 - 9 \Rightarrow 57$ (+ve)

$C_{23} - U_2 - V_3 = 64 - 0 - 34 \Rightarrow 30$ (+ve)

$C_{32} - U_3 - V_2 = 50 + 10 - 48 \Rightarrow 60 - 48 \Rightarrow 12$ (+ve)

$C_{34} - U_3 - V_4 = 30 + 10 - 9 \Rightarrow 40 - 9 \Rightarrow 31$ (+ve)

Conclusion :-

Since all the net values of net cost change relation are positive optimal solution is obtained

Hence, it is "optimal Allocation".

The total transportation cost for occupied cells is

$\Rightarrow$ RS $39 \times 23 + 10 \times 6 + 48 \times 8 + 9 \times 30 \times 0 \times 17 + 24 \times 16$

$\Rightarrow$ Rs $897 + 60 + 384 + 270 + 0 + 384$

$\Rightarrow$ RS 1995 /-

~~Unit-4~~

Assignment model :-

Jobs

$$
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & & j & & n \\
\hline
1 & C_{11} & C_{12} & C_{13} & \cdots & C_{1j} & \cdots & C_{1n} \\
2 & C_{21} & C_{22} & C_{23} & \cdots & C_{2j} & \cdots & C_{2n} \\
3 & C_{31} & C_{32} & C_{33} & \cdots & C_{3j} & \cdots & C_{3n} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
i & C_{i1} & C_{i2} & C_{i3} & \cdots & C_{ij} & \cdots & C_{in} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
n & C_{n1} & C_{n2} & C_{n3} & & C_{nj} & & C_{nn}
\end{array}
$$

LPP form of Assigment model :-

The Assignment model Can be started an

$$\text{Min Cost } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} * X_{ij}$$

$$S|to \quad X_{ij} = \begin{cases} 1 & \text{if} \\ 0 & \text{other wise} \end{cases}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \; [\text{If}$$

$$\sum_{i=1}^{n} X_{ij} = 1 \; [\text{If}$$

It is a special type of linear programming it deals. with allocation of ~~where~~ various origins to various and equal number of destinations [persons], one-to-one basics, such that to maximize the profit or to minimize the Cost

Mathematical form of Assignment model :-

Suppose there are 'n' jobs are to be performed or 'n' persons are avaliable for performing these jobs assume that every person Can do, one job at a time

If ith person is assigned with Z^{th} job, the solution is to find an assignment in such way that the total Cost of performing all job is the minimum

* If ith person is assigned with j^{th} job otherwise 0

$$X_{ij} = \begin{cases} 1 & \text{If} \\ 0 & \text{otherwise} \end{cases}$$

* If one job is done by ith person $\left\{ \sum_{j=1}^{n} x_{ij} = 1 \right\}$

* If only one person should be assigned to jth the job where x_{ij} the knowns that jth job is to be perform by ith person $\left\{ \sum_{i=1}^{n} x_{ij} = 1 \right\}$

Hungarian method :-

It is the simple and more efficient method to find the solution to the assignment Problem

[~~It is the simple~~ ~~to the given assignment~~ ~~model~~]

Ex :- man - machine
person - Region (Territory)

for minimization :-

The procedural steps as follows

Step 1 :- In the given matrix substract the smallest element in each row, from every element of that row. [row wise operations]

Step 2 :- In the redused matrix from the above step again substract the smallest element of a column from every of that Column [column wise operations]

Step 3 :- make assignment for this redused matrix as below

(a) check all rows successely until a row with exactly one zero is form. may an assignment to this single zero by drawing a square (□) around it Cross out (x) to all other zero's present in that Coressponding Column.

(b) Check all columns successely until a Column with single zero is formed. May an assignment to that single zero by drawing a square (□) around it Cross out (x) all other zero's present in the Corresssponding row

(c) Reapt the above steps, until all zero's in rows (or) Columns are either mark (or) crossed - out (x)

If the number of assignments may are equal to number of rows (or) columns [Being a square matrix], then optimal solution is obtained (or) reached otherwise go to the next step

Step 4 :- Draw the minimum number of horizontal and vertical lines to Cover all zero's in the redused matrix

The procedural steps as follows

a) MaRK (✓)

All rows do not have assignments

b) Mark (✓)

All Columns that have zero's in the marked row

c) Mark (✓)

mark all rows that have assignments in the marked Column

(d) Reapet above steps until there will be know rows (or) columns to be marked

(e) Draw lines to unmarked rows and marked : Columns.

Note :- Draw minimum number of lines to cover all zero's.

Step 5 :- If the number of lines draw is equal to number of rows (or) Columns the optimal solution is reached. other wise go the next step.

Step 6 :- select the smallest element among all uncovered elements substract these elements from all uncovered element and add it, to the elements that are present in the intersection of two lines

* Thus we obtain another redused matrix for fresh assignment.

Step 7 :- go to the step 3 rept the procedure until number of assignment made is equal to number of rows (or) Columns

<u>Note</u> :- Observe that every row (or) Column has an assignment then optimal Solution is obtained

problem :- 1 Solve the following

0	6	2	5
2	5	1	1
1	1	3	4
3	4	2	1

check the following condition

Number of rows = Number of columns

Then it is a balanced model.

By Hungarial method :-

Row wise operation

0	6	2	5
1	4	0	0
0	0	2	3
2	3	1	0

Column wise operation :

0	6	2	5
1	4	0	0
0	0	2	3
2	3	1	0

Select a single zero and make an assignment

$\boxed{0}$	6	2	5
1	4	$\boxed{0}$	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	2	3
2	3	1	$\boxed{0}$

Since the number of assignments is made equal to number of rows (or) colums optimal solution obtained

The total cost $0+1+1+1 = 3$ Rs/-

Problem :- 2

Find the optimal solution

$$\begin{bmatrix} 5 & 3 & 1 & 8 \\ 7 & 9 & 2 & 6 \\ 6 & 4 & 5 & 7 \\ 5 & 3 & 7 & 6 \end{bmatrix}$$

For minimization

Check the following condition

No of rows = no of columns

Hence the given problem is "Balanced model"

By Hungarian method

* Row wise operations :-

$$\begin{bmatrix} 4 & 2 & 0 & 7 \\ 5 & 7 & 0 & 4 \\ 2 & 0 & 1 & 3 \\ 2 & 0 & 4 & 3 \end{bmatrix}$$

* Column - wise operation

$$\begin{array}{cc} & \begin{array}{cccc} & & 2 & \end{array} \\ & \begin{bmatrix} 2 & 2 & \boxed{0} & 4 \\ 3 & 7 & \cancel{0} & 1 \\ \boxed{0} & \cancel{0} & 1 & \cancel{0} \\ \cancel{0} & \boxed{0} & 4 & \cancel{0} \end{bmatrix} & \begin{array}{c} -3 \\ -1 \\ \\ \\ \end{array} \end{array}$$

Case. I

Note :- The Problem 2 has multiple solutions.

Case II

2	2	$\boxed{0}$	4	3
3	7	$\cancel{0}$	1	1
$\cancel{0}$	$\boxed{0}$	1	$\cancel{0}$	
$\boxed{0}$	$\cancel{0}$	4	$\cancel{0}$	

Case I

(a)

1	1	$\boxed{0}$	3
2	6	$\cancel{0}$	$\boxed{0}$
$\boxed{0}$	$\cancel{0}$	2	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	5	$\cancel{0}$

Total Min Cost

Rs 1 + 6 + 6 + 3

Rs 16

(b)

1	1	$\boxed{0}$	3
2	6	$\cancel{0}$	$\boxed{0}$
$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$
$\boxed{0}$	$\cancel{0}$	5	$\cancel{0}$

Rs = 1 + 6 + 4 + 5

= 16

Hence the total minimum Cost = 16

II sem supple May 2020

Problem :- Assign the tasks to persons which minimises the time for the following data

	P	Q	R	S	T
A	1	3	5	7	9
B	4	7	10	13	16
C	8	12	16	20	24
D	13	18	23	28	33
E	19	25	31	37	43

for minimization

check the following condition

total rows = total columns

Hence the given problem is " Balanced mode ".

By Hungarial method

Row wise operation :-

$$
\begin{bmatrix}
0 & 2 & 4 & 6 & 8 \\
0 & 3 & 6 & 9 & 12 \\
0 & 4 & 8 & 12 & 16 \\
0 & 5 & 10 & 15 & 20 \\
0 & 6 & 12 & 18 & 24
\end{bmatrix}
$$

Column wise operations :-

intersection value 1

$$
\begin{array}{c}
2 \\
\begin{bmatrix}
\cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \\
\boxed{0} & 1 & 2 & 3 & 4 \\
\cancel{0} & 2 & 4 & 6 & 8 \\
\cancel{0} & 3 & 6 & 9 & 12 \\
\cancel{0} & 4 & 8 & 12 & 16
\end{bmatrix}
\end{array}
\begin{array}{l}
\\ \\ 3 \\ -1 \\ -1 \\ -1
\end{array}
$$

there is no single zero in first column

mark all rows ~~that~~ don't have assignments

$$
\begin{array}{c}
2 \\
\begin{bmatrix}
1 & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \\
\cancel{0} & \boxed{0} & 1 & 2 & 3 \\
\boxed{0} & 1 & 2 & 5 & 7 \\
\cancel{0} & 2 & 5 & 8 & 11 \\
\cancel{0} & 3 & 7 & 11 & 15
\end{bmatrix}
\end{array}
\begin{array}{l}
\\ \\ \\ - 3 \\ \rightarrow 1 \\ \rightarrow 1
\end{array}
$$

$$\begin{bmatrix} 2 & \cancel{0} & \cancel{0} & 0 & 0 \\ 1 & \cancel{0} & 0 & 1 & 2 \\ \cancel{0} & \boxed{0} & 1 & 4 & 6 \\ \boxed{0} & 1 & 4 & 7 & 10 \\ \cancel{0} & 2 & 6 & 10 & 14 \end{bmatrix}$$

Un - Balanced assignment model :-

If the no of rows of columns are not equal then it is known as un Balanced model to balance we introduce a dummy row (or) Column to balance the given problem we assign Zero Cost for every Cell Indroduced (or) Added newly to the row or Column. Then it Can be Solved by the models

problem :-

$$\begin{bmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \end{bmatrix}$$

Sol check the following Condition

No of rows = No of Columns

The given problem is an " un - Balanced model "

The problem is added with a row

The Balanced model is as follows.

$$\begin{bmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\cancel{24}$ 18 / 6

for minimization

By Hungarian method

Row wise Operations

$$\begin{bmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Column wise operation

$$\begin{array}{c} ✓2 \\ \begin{bmatrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 9 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix} \end{array} \begin{array}{c} ✓3 \\ ✓3 \\ ✓3 \\ \end{array}$$

$$\begin{array}{c} 2 \quad\quad 2 \\ \begin{bmatrix} \boxed{0} & 1 & 5 & 9 \\ \cancel{0} & \boxed{0} & 4 & 6 \\ \cancel{0} & \cancel{0} & 4 & 7 \\ 5 & \cancel{0} & \boxed{0} & \cancel{6} \end{bmatrix} \end{array} \begin{array}{c} -3 \\ -3 \\ -1 \\ \end{array}$$

$$\begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 9 & 4 & 0 & 0 \end{bmatrix}$$

Note :- the problem has multiple solutions

(a)

$$\begin{bmatrix} \boxed{0} & 1 & 1 & 5 \\ \cancel{0} & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & \boxed{0} & 3 \\ 9 & 4 & \cancel{0} & \boxed{0} \end{bmatrix}$$

(b)

$$\begin{bmatrix} \boxed{0} & 1 & 1 & 5 \\ \cancel{0} & \cancel{0} & \boxed{0} & 2 \\ \cancel{0} & \boxed{0} & \cancel{0} & 3 \\ 9 & 4 & \cancel{0} & \boxed{0} \end{bmatrix}$$

the no of assignment = no of rows (or) columns
the optimal solution is obtained (or) reached
" Hence it is the the optimal assignment "

(a) The Total (minimum) cost = Rs 50/-

(b) The Total (minimum) Cost = Rs 50/-

For maximization :-

Sometimes we may have to deal with an assignment model in which the objective to maximize problems (or) maximize the sales (or) maximize the total output of the machines.

for these problems substract all elements of the matrix from the largest element in that matrix then the problem is converted into minimization. Then apply hungarian method.

problem :- Solve the problem for maximization

16	10	14	11
14	11	15	15
15	15	13	12
13	12	14	15

Sol :-
The given Problem is for maximization Convert it into "minimization"

select the largest element in the matrix and substract all the elements from it

For minimization :-

0	6	2	5
2	5	1	1
1	1	3	4
3	4	2	1

check the following Condition
no of rows = no of columns
By "Hungarian method".

Row wise operation

$\boxed{0}$	6	2	5
1	4	$\cancel{0}$	$\boxed{0}$
$\cancel{0}$	$\boxed{0}$	2	3
2	3	1	$\boxed{0}$

Column wise operation

$\boxed{0}$	6	2	5
1	4	$\boxed{0}$	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	2	3
2	3	1	$\boxed{0}$

The Total (maximum) cost = Rs 16 + 15 + 15 + 15
= Rs 61

problem :-
Solve the problem for maximization

20	15	25	25	29
13	19	30	13	19
20	17	14	12	15
14	20	20	16	24
14	16	19	11	22

The given problem is for maximization
Convert into the "Minimization"
Select the largest element in the matrix and
Substract all the element from it

for minimization :-

10	15	5	5	1
17	11	0	17	11
10	13	16	18	15
16	10	10	14	6
16	14	11	19	8

Check the following Condition

no. of Rows = no of Columns

By " Hungarian method

Row wise operation

9	14	4	4	0
17	11	0	17	11
0	3	6	8	5
10	4	4	8	0
8	6	8	11	0

Column wise operation

9	11	4	$\boxed{0}$	$\cancel{0}$
17	8	$\boxed{0}$	13	11
$\boxed{0}$	$\cancel{0}$	6	4	5
10	1	4	4	$\boxed{0}$
8	3	3	7	$\cancel{0}$

8	10	4	$\boxed{0}$	1
16	7	$\boxed{0}$	12	12
$\boxed{0}$	$\cancel{0}$	5	3	6
9	$\boxed{0}$	3	3	$\cancel{0}$
7	2	2	6	$\boxed{0}$

The total maximization Cost = Rs 25 + 30 + 20 + 20 + 22

= Rs 117 /-