Unit - 4

Game Theory

PAY-OFF MATRIX OF PLAYER - A

Player B

$$
\begin{array}{c|cccccccc|}
 & 1 & 2 & 3 & \cdots & j & \cdots & n \\
\hline
1 & V_{11} & V_{12} & V_{13} & \cdots & V_{1j} & \cdots & V_{1n} \\
2 & V_{21} & V_{22} & V_{23} & \cdots & V_{2j} & \cdots & V_{2n} \\
3 & V_{31} & V_{32} & V_{33} & \cdots & V_{3j} & \cdots & V_{3n} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
i & V_{i1} & V_{i2} & V_{i3} & \cdots & V_{ij} & \cdots & V_{in} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
m & V_{m1} & V_{m2} & V_{m3} & \cdots & V_{mj} & \cdots & V_{mn} \\
\hline
\end{array}
$$

Player A

PAY-OFF MATRIX of PLAYER - B

Player B

$$
\begin{array}{c|cccccccc|}
 & 1 & 2 & 3 & \cdots & j & \cdots & n \\
\hline
1 & -V_{11} & -V_{12} & -V_{13} & \cdots & -V_{1j} & \cdots & -V_{1n} \\
2 & -V_{21} & -V_{22} & -V_{23} & \cdots & -V_{2j} & \cdots & -V_{2n} \\
3 & -V_{31} & -V_{32} & -V_{33} & \cdots & -V_{3j} & \cdots & -V_{3n} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
i & -V_{i1} & -V_{i2} & -V_{i3} & \cdots & -V_{ij} & \cdots & -V_{in} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
m & -V_{m1} & -V_{m2} & -V_{m3} & \cdots & -V_{mj} & \cdots & -V_{mn} \\
\hline
\end{array}
$$

Player A

$= 0$ (zero)

In theory is one type of descision making theory the choice of action is determined after Considering all possible alternatives available for playing the game, rather than based on the possiabilities of serveral outcomes or outputs.

* Creterion

The Creterion implies the assumption of Rationality to act as — " to maximize a person minimum gain (MAXMINI)" or — " TO minimize a persons maximum loss (MINIMAX)".

*GAME :-

It may be defined as an activity between two ~~one~~ (or) more persons involving activities by each person, according to a set of rules, at the end of which every person receives some benefit (or) loss (Quantitative)

*PAY-OFF :-

It is the Quantitative measure of output (writtens) a person receives at the end of each game

*ZERO-SUM GAME :-

If the players may payments only to each other, the loss of one player — "Equal to the gain of other player" and nothing comes from outside than that gain is known as zero-sum Game "The Net gain for the game is zero"

*Non-zero-sum Game :-

The loss of one player is not equal to the gain of other player. The net gain for the gain is not equal to zero then that gain is known as Non-zero-sum Game.

*STRATEGY :-

There are two types of strategy for a game

a) Pure strategy :-

If a player knowns exactly what the other player is going to do then a determina situation is resultant or generater the objective is maximize or minimize the losses or gains · pure one type of decision making in the game to select a particular course of action

b) Mixed strategy :-

If a player is Guessing as to which activity or decision is to be selected than other in a situation of the gain · A player doesn't have Complete knowledge (or) full about the strategy of player team

* Saddle point [Equallibrium point]

The saddle point of the pay off matrix is the position of an element in the matrix, which is the minimum in its row and maximum in its Column

→ Mathematically if a pay off matrix $\{v_{ij}\}$

$$\max_i [\min_j (v_{ij})] = \min_j [\max_i (v_{ij})] = V_{r,s}$$

Then the matrix is set to have a saddle point.

* optimal strategy :-

If the pay off matrix $\{V_{ij}\}$, has the saddle point (r, s) then the players are set to have, r^{th} & s^{th} Strategy (optimal strategy) respectively

* Value of game :-

The pay of matrix $\{v_{ij}\}$ at the saddle point (r, s) is known as value of the game it is obviously equal to MAXMINI ($\underline{V}$) and MINIMAX ($\overline{V}$) of the game

* Two_person-zero-sum and Game :-

[Rectangular game] a Game with only two players (Player A & player B) is known as two-person-zero-sum and Game.

when the loss of one person is equal to the gain of other player such that the net gain of the game is zero. The players may not have equal strategy Hence it is a Rectangular Game.

* PAY OFF MATRIX FORM :-

Suppose player A has m activities and player B has n activities then a pay off matrix can be form by the following rules

(a) The Row values in the matrix are the returns available to the player A Game.

(b) Column value in the matrix are the returns available to the player B

(c) The cell entry V_{ij} is the payment to the player A in the pay off matrix of the player A. when player A selects an activity i and player B selects an activity j.

(d) In the zero sum two person game the cell entry in pay off matrix of the player B will be negative with respect to cell entry in the pay off matrix of Player A.

The sum of pay off matrices for player A and player B is equal to zero

Ex :- TOSSING OF A COIN (TWICE)

PAY-OFF MATRIX OF PLAYER-A

		Player-B	
		T	H
player-A	H	−1	+1
	T	+1	−1

It is a pure strategy a coin is tossed a twise every player has its choice between two pure strategy [head and tail] and matrix is in the following (2 X 2) form

Game with Saddle point :-

maximini and minimax Criterion :-

If the maxmini & minimax value are equal then the game is said to have " SADDlE POINT ". the Corresponding pay-off value is known as value of the game $[\underline{V} = \overline{V} = V]$

The procedural steps as follows .

1. Note down the minimum pay-off an value for each Row it is denoted by the circle 'O'
2. Select the maximum pay-off value in each Column and it is denoted by the symbol " □ " (square)

If the MAXMINI and MINIMAX values are equal then mark the Position of the pay off value in the matrix. This value represents the value of the game it is also known as SADDLE POINT of the matrix .

problem :-

player - B

player-A	I	II	III	IV
I	1	7	3	4
II	5	6	4	5
III	7	2	9	3

the given matrix is assumed to be pay-off matrix of the PLAYER A .

Player A is maximizing agent . The object is to maximize minimum gains

Player B is minimizing agent is controlled or checking the strategy of player A

		Player B I	II	III	IV	
	I	1	7	3	4	1
Player A	II	5	6	4	5	4
	III	7	2	0	3	0
		7	7	4	5	

Since, the matrix has Row minimum value and Column matrix maximum value all equal in matrix

The model has a saddle point.

value of Game $V = 4$

(maxmini = minmax)

optimal strategis of player-A (I, II, III) = (0, 1, 0)

optimal strategis of player-B (I, II, III, IV) = (0, 0, 1, 0)

Game without saddle point :-

Such models can be slove by two methods.

1. Graphical method
2. Arithmetic method

1. Graphical method :-

The graphical method is useful for the game where the pay-off matrix (or) the model / problem is of a size $(2 \times n)$ (or) $(m \times 2)$.

The game with mixed strategy for one of the players in the two person zero sum game optimal strategy for the both the players non-zero probability if one player has only two strategis the other will also utilize the maximum no. of strategies.

Hence this method is useful for optimal strategies

For $(2 \times n)$ Game model :-

The "highest point" of lower boundary (or) ~~envelo~~ envelope will give maximum expected pay off, among the minimum expected pay off on the lower boundary. the optimal value of propability P_1 & P_2 will be found. Now the two strategies of player B Corresponding the those lines, which pass to the maximum point can be determined. It is in the redused matrix of the game (2×2) model.

For $(m \times 2)$ Game model :- (solved as)

It is also ~~Slove~~ treated as in the Same method here the "upper boundary" (Boudary) of the stright lines Corresponding to the expected pay off matrix of player B. It will be maximum expected payoff to the player B and "lowest point" on this bound will provide the minimum expected pay-off (min max) and optimal values of propabilities Q_1 & Q_2

2. Arithmetic method :-

This method provides a simple technique to find the optimal strategy for each Player (2×2) method redused matrix, without Saddle point

The procedural step as follows.

1. find the difference b/w two values in the Column and place it under Column$_2$ neglecting negative sign (take absolute values)
2. find the difference b/w two values in the Column$_2$ and place it under Colum$_1$ neglecting negative sign [take absolute value]

3. Repeat the above steps for the two rows also the values does uptain are known as "ODDMENTS" they are the frequency with which the player must utilize their strategy or activities to get the optimal value

problem 2 :-

Player B

		I	II	III
player A	I	1	3	11
	II	8	5	2

The given matrix is assumed to be pay-off matrix of player A

player A is maximizing agent"

the objective is to maximize minimum gains

player B is minimizing agent. He wants to check strategy of player A. He wants to minimize maximum loss

Player B

		I	II	III		Row Min
player A	I	1	3	11	(M)	1
	II	8	5	2	(N)	2
Column max		8	5	11		

There is No "Saddle point"

By graphical method

The given problem belongs to (2 x n) Game model

(maxmize - minimum Game)

By graphical method
the given game problem is values to (2x2) model as follows

		PL-B			oddments
		II	III		
PL-A	I	3	11	3	3/11
	II	5	2	8	8/11
		9	2	11	

$3\times9=27+11\times2=22$
$27+22=49$

$5\times9=45+2\times2=4$
$=49$

oddments $9/11$ $2/11$

Conclusion :-

The optimal strategy of player-A (I II) = $(3/11 ; 8/11)$

The optimal statement player-B (I, II, III) = $(0, 9/11, 2/11)$

Value of Game $(\underline{v}) = \dfrac{(3\times9)+(11\times2)}{9+2}$

$= \dfrac{49}{11}$

27
27

Problem :- 3

		Player-B		Row minimum
		I	II	
player A	I	2	4	2
	II	2	3	2
	III	3	2	2
	IV	-2	6	-2
Column max		3	6	

It is assume that the given problem is a pay-off matrix of player A. Player A is
The objective is to maximize minimum gains
Player B is checking the strategys of player A
Player B is minimizing agent
The objective is minimize maximum loss
There is " No saddle point"
The given problem belongs to (mx2) model.

The given problem belongs to ~~(2x2)~~ (2X2) model as follows

		PLB			
		I	II		oddments
PL-A	I	2	4	1	1/3
	III	3	2	2	2/3
		2	1	3	
	oddments	2/3	1/3		

Conclusion :-

The optimal strategy of playerA(I II III IV) = $(\frac{1}{3}, 0, \frac{2}{3}, 0)$

The optimal statement player B (I, II) = $(\frac{2}{3}, \frac{1}{3})$

Value of Game $(\bar{V}) = \frac{(3\times 2)+(2\times 1)}{2+1}$

(MINIMAX)

$$= \frac{6+2}{3}$$

$$= \frac{8}{3}$$

Problem :- 4 MCA NOV 22

use the graphical method in solving the following game and find the value of the game

		Pl-B				
		I	II	III	IV	
Pl-A	I	2	2	3	-2	-2
	II	4	3	2	6	2
		4	3	3	6	

m rows
n column

The given matrix is assumed to be pay-off matrix of player A

Player A is maximizing agent

The objective is to maximize minimum gains

Player B is minimizing agent

Player B checking the strategy of player A

The objective is minimize Maximum Loss

There is "No saddle point"

By graphical method

The given problem belongs to $(2 \times n)$ Game model

		Pl-B III	IV		oddments
Pl-A	I	3	-2	4	4/9
	II	2	6	5	5/9
oddments		8	1	9	
		8/9	1/9		

$3+2=5$
$6-2=4$
$6+2=8$
$3-2=1$

Conclusion :-

The given Optimal strategy of player A (I II III IV) $= \left(\frac{4}{9}, \frac{5}{9}, 0, 0\right)$

The optimal statement player B (III, IV) $= \left(\frac{8}{9}, \frac{1}{9}\right)$

$$\text{Value of Game } (\bar{V}) = \frac{(2\times8)+(6\times1)}{8+1} = \frac{16+6}{9} = \frac{22}{9}$$

Problem :- 5 MBA - Nov 2020

	I	II	
I	1	2	1
II	5	4	4
III	-7	9	-7
IV	-4	-3	-3
V	2	1	1
	5	9	

The Given problem is a payoff matrix of player A

Player A is a maximizing Agent.

The objective is a Maximize minimum gains

Player-B is checking the strategy of player-A

Player-B is minimizing Agent

There is "no saddle point".

By the graphical method

The given problem belongs to (m X 2) model

		Player B I	II		oddments
Player A	II	5	4	16	16/17
	III	−7	9	1	1/17
		5	12	17	
oddments		5/17	12/17		

$\frac{-35+108}{5+12}$ $\frac{73}{17}$

Conclusion:-

The optimal strategy of player A (I II III IV) = (0, 16/17, 1/17, 0)

The optimal statement of player B (I, II) = (5/17, 12/17)

Value of Game ($\bar{V}$) $= \frac{(-7\times5)+(9\times12)}{5+12}$

$= \frac{-35+108}{17}$

$= \frac{73}{17}$

problem:-6

MBA NOV 2020

7)

(b) Game value for the following data

Playe A	Player B B1	B2
A1	5	3
A2	2	4

		I Player B	II	Row min
Player A	I	5	3	3
	II	2	4	2
Colum max		5	4	

The given matrix is assumed to be pay-off matrix of player A

player A is maximizing agent

the objective is to maximize minimum gains

player B is minimizing agent

The objective is minimize Maximum Loss

player B checking the strategy of player A

There is "No Saddle point" ∴ Given problem belongs to (2×2) model.

		Player B I	Player B II		oddments
player A	I	5	3	2	2/4
	II	2	4	2	2/4
		1	3		
oddments		1/4	3/4		

$q_1 = \frac{1}{1+3} = \frac{1}{4}$ $q_2 = \frac{3}{1+3} = \frac{3}{4}$

$P_1 = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$

$P_2 = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$

Conclusion :-

The optimal strategy of player A $(I, II) = \left(\frac{1}{2}, \frac{1}{2}\right)$

$= \frac{1}{2} + \frac{1}{2} \Rightarrow \cancel{\frac{2}{2}} \Rightarrow 1$

The optimal statement of player B $(I\ II) = \left(\frac{1}{4}, \frac{3}{4}\right)$

$= \frac{1}{4} + \frac{3}{4} \Rightarrow \frac{4}{4} \Rightarrow 1$

Value of Game $(v) \Rightarrow \frac{(5\times1)+(3\times3)}{1+3} \Rightarrow \frac{5+9}{4}$

$= \frac{14}{4}$

$$\left\{\begin{array}{l}
\Rightarrow \frac{(2\times1)+(4\times3)}{1+3} \Rightarrow \frac{2+12}{1+3} = \frac{14}{4} \\
\Rightarrow \frac{(5\times2)+(2\times2)}{2+2} \Rightarrow \frac{10+4}{4} = \frac{14}{4} \\
\Rightarrow \frac{(3\times2)+(4\times2)}{2+2} \Rightarrow \frac{6+8}{2+2} = \frac{14}{4}
\end{array}\right.$$

MBA - NOV 22

problem :- 7

Throw some light on the significance of game theory for managers in a business organization. Solve the following 2 person zero sum game with the following 3X2 pay off matrix of player A

		Player B	
		B_1	B_2
playerA	A_1	9	2
	A_2	8	6
	A_3	6	4

Ans :-

	Player B B_1	B_2	Row min
A_1	$\boxed{9}$	2	2
playerA A_2	8	$\boxed{6}$	6
A_3	6	4	4
Colum max	9	6	

The given matrix is assumed to be pay-off matrix of the Player A

Player A is maximizing agent

The objective is to maximize minimum gains

Player B is minimizing agent

Playe B is checking the strategy of player A

Since the Row minimum value and column maximum value are equal

The model has a "Saddle point"

Value of Game (V) = 6 (MAXMINI = MINIMAX)

optimal strategy of player A (A_1 A_2 A_3) = (0,1,0)

optimal strategy of player B (B_1, B_2) = (0,1)

MBA - 2028 problem :- 9

Find game to players A & B giving the pay off matrix of player A

Player B

		B_1	B_2	B_3	
player A	A_1	$\boxed{12}$	-8	-2	-8
	A_2	6	$\boxed{7}$	$\boxed{3}$	3
	A_3	-10	-6	2	2
	Colum max	12	7	3	

The given matrix is assumed to be pay-off matrix of the player A

player A is maximizing agent

The objective is to maximize minimum gains

player B is minimizing agent

player B is checking the strategy of player A

since the Row minimum value and Column maximum value are equal

the model has a " saddle point"

Value of Game (V) = 3 (Maxmini = minimax)

optimal strategy of player A $(A_1, A_2, A_3) = (0, 1, 0)$

optimal strategy of player B $(B_1, B_2, B_3) = (0, 0, 1)$

Analytical method :-

Domiance Rule properity (or) for (m x n) matrix :-

If the pay off matrix of a game model can be redused by eliminating forces of action or strategy or game plan that is less importance to another strategy such strategy are set be Dominated by other strategies.

General rule (or) procedural steps :-

1. If all elements of a row (say k^{th} row) or less than or equal to corresponding elements of another row (say r^{th} row), then k^{th} row is dominated by r^{th} row.
2. If all elements of a column (say k^{th} column) are greather or equal to corresponding elements of another column (say r^{th} colum), then k^{th} Delete the dominated rows or colum.
3. Delete the dominated rows or column.
4. If few linear combinations (Average) of some rows dominates i^{th} row, then i^{th} row can be deleted
5. If few linear combinations (Average of some columns dominates j^{th} column, then j^{th} column can be deleted

Note :-

1. ~~such~~ first search for common numerical values present in rows or columns
2. Row wise : RETAIN higher values
3. Column wise : RETAIN lower values

problem :- 1

	I	II	III
I	1	7	2
II	6	2	7
III	5	1	6

The given matrix is assumed to be pay-off matrix of the player A

player A is maximizing agent

the objective is to maximize minimum gains

player B is minimizing agent

player B checking the strategy of player A

The objective is minimize maximum loss

There is "No Saddle point"

PL-B

		I	II	III	Row minimum
PL-A	I	1	7	2	1
	II	6	2	7	2
	III	~~5~~	~~1~~	~~6~~	1
Column maximum		6	7	7	

By analytical method. By Domiance Rule method.

R_2 values are less than R_3 values they are

Dominated by R_2 values

Hence R_3 Can be deleted.

The Redused as below.

PL-B

		I	II	III
PLA	I	1	7	2
	II	6	2	7

3rd Column less 1st Column

C_3 values are higher than C_1 values

C_3 is dominated by C_1

Delete C_3 values

The Reduced matrix has below

PL-B

	I	II	oddments	
I	1	7	4	4/10
II	6	2	6	6/10
oddments	5	5	10	
	5/10	5/10		

Conclusion :-

The optimal staregy's of player A (I, II, III) = (4/10, 6/10, 0)

optimal staregy's of player B (I, II, III) = (5/10, 5/10, 0,)

value of the Game (v) = ($\underline{V} = V = \overline{V}$)

$$= \frac{(6\times5)+(8\times5)}{5+5}$$

$$= \frac{30+10}{10}$$

$$= \frac{40}{10} = 4$$

---

problem :- 2

PL-B

		I	II	III	Row min
PL-A	I	4	2	4	2
	II	2	4	0	0
	III	4	0	8	0
column max		4	4	8	

The given matrix is assumed to be pay off matrix of the player-A

Player A is maximizing Agent, the objective is to maximize minimum gains

Player B is minimizing Agent

The objective is minimize maximum loss

player B is checking the strategy of player A

there is "No Saddle point"

By Analytical method

By Dominonce Rule method

The Average of R_2 and R_3 values is less than are one value

Hence R_2, R_3 Can be deleted

Now Redused matrix as follows

	I	II	III
I	4	2	4

C_1 and C_3 values are higher than C_2 value they are dominated by C_1 value

Hence they Can be deleted

The Reduced matrix has follows

	II
I	2

The optimal staregy's of player A (I, II, III) = (1, 0, 0)

The optimal staregy's of player B (I, II, III) = (0, 1, 0)

Value of the Game (V) = 2

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problem :- 3

3	5	4	9	6
5	6	3	7	8
8	7	9	8	7
4	2	8	5	3

The given matrix is is assumed to the pay off matrix of player A

player – A is maximizing agent

player – B is minimizing agent

player – A objective of a maximize to minimize gains

player – B is checking strategy ~~of~~ a player – A

The objective of a minimize to maximize loss

	I	II	III	IV	V	Row minimum
I	3	5	4	$\boxed{9}$	6	3
II	5	6	3	7	8	3
III	$\boxed{8}$	$\boxed{7}$	$\boxed{9}$	8	$\boxed{7}$	7
IV	~~4~~	~~2~~	~~8~~	~~5~~	~~3~~	2
Column maximum	8	7	9	9	7	

This is model has a "saddle point".

By Analytical method

By Dominated method

R_3 values are less-than R_4 values

there are dominated by R_3 values

Hence R_4 values can be Deleted

The redused matrix is

	I	II	III	IV	V
I	3	5	4	9	6
II	5	6	3	7	8
III	8	7	9	8	7

C4 value are higher than C2 values
C4 dominated by C2 values Hence C4 Deleted
The Reduced matrix as bellow

	I	II	III	IV
I	3	5	4	6
II	5	6	3	8
III	8	7	9	7

C4 values are higher than C2 values
C4 Dominated by C2 values Hence C4 Deleted. Reduced matrix below

	I	II	III
I	3	5	4
II	~~5~~	~~6~~	~~3~~
III	8	7	9

R3 values are less than R2 values
there are Dominated by R3 values. R2 Deleted

	I	II	III
I	~~3~~	~~5~~	~~4~~
II	8	7	9

R2 values are less than R1 values
There are Dominated by R2 values
Hence R1 values are Deleted

	I	II	III
I	~~8~~	7	~~9~~

C1 and C3 values are higher.
Hence C2 values they are Dominated by C2 values
C1 & C3 Can be deleted

	II
I	7

Conclusion :-

optimal solution player A = (I, II, III, IV)
= (0, 0, 1, 0)

optimal solution playerB = (I, II, III, IV, V)
= (0, 1, 0, 0, 0)

values of Game = 7

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problem :- 4

	I	II	III	IV	V
I	2	4	3	8	4
II	5	6	3	7	8
III	6	7	9	8	7
IV	4	2	6	4	3

The given matrix is assumed to the pay off matrix
Player A is maximizing agent
Player B is minimizing agent
player A is objective of maximize to minimize gain
Player B is checking strategy for a the objective of a minimize maximize loss

	I	II	III	IV	V	
I	~~2~~	~~4~~	~~3~~	~~8~~	~~4~~	2
II	5	6	3	7	8	3
III	$\boxed{6}$	7	9	8	7	6
IV	~~4~~	~~2~~	~~6~~	~~4~~	~~3~~	2
	6	7	9	8	8	

This is " Saddle point "

By Analytical method

By Dominated method

R_3 values higher than R_1 and R_4

R_1 and R_4 values are dominated by R_3 values

Hence it Can be eleminated R_1 & R_4 Values

Now Can be Reduced matrix

	I	II	III	IV	V
II	5	6	3	7	8
III	6	7	9	8	7

C_1 values are lower than C_2 and C_4, C_5 values

C_4, C_2, C_5, are dominated by C_1 values

Hence C_2, C_4, C_5 Deleted

Reduced matrix

	I	III	
II	~~5~~	~~3~~	3
III	6	9	2
	6	8	

R_3 values are higher than R_2 values

R_2 values dominated

Hence R_2 values are deleted

	I	II
III	6	9

	I
III	6

optimal strategy of player A

(I, II, III, IV) = (0, 0, 1, 0)

Play B (I, II, III, V) = (1, 0, 0, 0)

Value of Game 6

	I	II	III	IV	V
I	~~2~~	~~4~~	~~3~~	8	4
II	~~5~~	~~6~~	~~3~~	7	8
III	$\boxed{6}$	7	9	8	7
IV	~~4~~	~~2~~	~~6~~	4	3

This is "saddle point"

$\Leftarrow$ process II

Same answer

C_1 vs C_4 C_4 Deleted

C_2 vs C_5 C_5 Deleted

R_3 vs R_2, R_4, R_1 Deleted

C_1 vs C_2 C_3 Deleted

The Reduced matrix was a single matrix

	I
III	6

Value of Game = 6

player A : (I , II , III , IV) : (0, 0, 1, 0)
Player B : (I , II , III , IV , V) : (1, 0, 0, 0, 0)

Problem :-5

	I	II	III	IV	V	VI	
I	0	0	~~0~~	~~0~~	~~0~~	0	0
II	4	2	~~0~~	~~2~~	1	1	0
III	4	3	1	3	2	2	1
IV	4	3	7	-5	1	2	-5
V	4	3	4	-1	2	2	-1
VI	4	3	~~3~~	~~2~~	~~2~~	2	-2
	4	3	7	3	2	2	

There is no " saddle point "

C_5 values C_1, C_2, C_6 value are higher than C_5 values Hence C_1, C_2, C_6 Deleted

R_3 values higher than R_1, R_2 values R_1, R_2 Dominted by values R_3 Hence R_1, R_2 Deleted

R_5 values are Higher than R_6

R_6 values dominated by R_5

Deleted R_6

	III	IV	V
III	1	3	2
IV	7	-5	1
V	~~4~~	~~-1~~	2

The linear Average of C_3 and C_4 values lower then C_5. Hence C_5 is dominated by average values C_4, C_3 Hence C_5 Can be deleted

The linear average of R_3 and R_4 is equal to R_5
Hence delete R_5 values

	III	IV		
III	1	3	-12	-12/14
IV	7	-5	2	2/14
	8	6	14	
	8/14	6/14		

The optimal solution strategy of play A = (I, II, III, IV, V, VI)
$= (0, 0, 8/14, \frac{6}{14}, 0, 0)$

The optimal Solution strategy of playe B = (I, II, III, IV, V, VI)
$= (0, 0, \frac{12}{14}, \frac{2}{14}, 0, 0)$

$\bar{V} = \frac{8+18}{14} = \frac{26}{14}$

problem :- 6
H.W

	I	II	III	IV
I	8	10	9	14
II	10	12	8	12
III	13	12	14	13

The given matrix Assumed to the pay-off matrix of player-A
Player-A is minimizing agent the objective to maximize and minimize gains.
Player-B is checking strategy of Player-A
Player-B is minimizing Player-A

		I	II	III	IV	Pl-B
Pl-A	I	8	10	9	14	8
	II	~~10~~	~~12~~	~~8~~	~~12~~	8
	III	13	$\boxed{12}$	14	13	12
		13	12	14	14	

This is "saddle point"

By Analytical method

By Dominance Rule

The R_3 values less than R_2 values

there are dominated by R_3 values

R_2 values deleted

The reduced matrix (4x2)

	I	II	III	IV
I	8	10	9	14
III	13	12	14	13

C_4 values is higher than C_1, C_2 values

dominated by C_1, C_2

C_4 values can be deleted

The resultant matrix (3x2)

	I	II	III
I	~~8~~	~~10~~	~~9~~
III	13	12	14

R_3 values are less than R_1 values

they dominated by R_3 values

R_1 values deleted

The Resulted matrix (1x3)

	I	II	III
III	13	12	14

C_1, C_3 values higher than C_2 values

C_1, C_3 dominated by C_2

Hence C_1, C_3 deleted

The resulted matrix (1x1)

	II	PL - B
pl- A III	12	

The optimal strategy player-A = (I, II, III) = (0, 0, 1)
the optimal strategy Player-B = (I, II, III, IV) = (0, 1, 0, 0)

Game value V = 12

problem :-7

	I	II	III
I	12	-8	-2
II	6	7	3
III	-10	-6	2

The Solve matrix assumed to the pay-off matrix of player A
player A is minimizing agent the objective is to maximize and minimize gains
player-B is checking strategy of player - A
Player-B is minimizing player - A

	I	II	III	
I	12	-8	-2	-8
II	6	7	3	3
III	10	-6	2	-10
	12	7	3	

there is a " Saddle point "
By Analytical method
By Dominance method
R_2 values less than R_3 values
They are dominated by R_2 values
Hence R_3 Deleted

	I	II	III
I	~~12~~	~~8~~	~~2~~
II	~~6~~	7	3

C_1 values are higher than C_3 values
they dominated C_3 value
Hence C_1 values deleted
R_1 Values are less than R_2 values
Hence delete R_1

	II	III
II	7	3

C_2 Higher the C_3
C_3 dominated by C_2
Hence C_2 deleted

	III
II	3

Game value 3
The optimal strategy player-A (I, II, III) = (0, 1, 0)
The optimal strategy player-B (I, II, III) = (0, 0, 1)

Problem :- 8

	I	II	III	IV	
I	19	6	7	5	6
II	7	3	14	6	3
III	12	8	18	4	4
IV	8	7	13	-1	-1
	19	8	18	6	

The is "NO Saddle point"
By Analaytical method
By Dominated method

	I	II	III	IV	
I	19	6	7	5	6
II	7	3	14	6	3
III	12	8	18	4	4
IV	~~8~~	~~7~~	~~13~~	~~-1~~	-1
	19	8	18	6	

R_3 Vs R_4

	I	II	III	IV
I	19	6	7	5
II	7	3	14	6
III	12	8	18	4

C_2 Vs C_1, C_3 deleted

	II	IV
I	~~6~~	~~5~~
II	~~3~~	~~6~~
III	8	4

	II	IV
III	~~8~~	4

	IV
III	4

optimal solution of playerA (I II III) = (0, 0, 1)

optimal Solution of player B (I, II, III, IV) = (0, 0, 0, 1)

Game value 4