

3. Network Techniques :-

A Network Consists of set of Nodes (vertices) and a set of Arcs each node represents a location (city) and each Arc represents the connections [road lines], between two different locations (city's). The number on each Arc represents the distance between two locations. There are three types of Network techniques

1. Shortest path method :-
2. Minimum Spanning tree model
3. Maximum flow model

The shortest path model can be further divided into two types they are (a) shortest path b/w only one pair of nodes (b) shortest path b/w any pair of nodes in the network.

Systematic method and Dijkstra's Algorithm :-

They can be applied to find the shortest path between one pair of nodes in the network.

floyd's algorithm :-

This technique can be applied to find the shortest path b/w any pair of nodes in the network.

Prim's algorithm and Kruskal's algorithm :-

These techniques can be applied to find the solution for minimum spanning tree.

Shortest path method :-

The objective of transportation problem of an organization is to find the shortest path to a specified node from the other nodes in the network (Road connectivity), for shipping cargo or materials merchandise. The shortest path model to develop to find the solution.

The following methods are applied to find the shortest path in the distance matrix. The following techniques are applied

- a. Systematic method
- b. Dijkstra's algorithm
- c. Floyd's algorithm

(a) systematic method :- The procedural steps are as follows

Step 1 :- Represents the details of Distance network in the form a table. (say table 1)

To each node a separate column is provided the Arc's connecting lines with Distances, that are emerging (or) coming out emanating from the selected node are arranged in the increasing order. (Ascending order), from top to bottom.

Step 2 :- Select the node 1 and set the Cumulative distance covered up to this node, to zero and show at the top of the table 1

Step 3 :- Delete all Arc's [entering lines] that are pointing towards node 1.

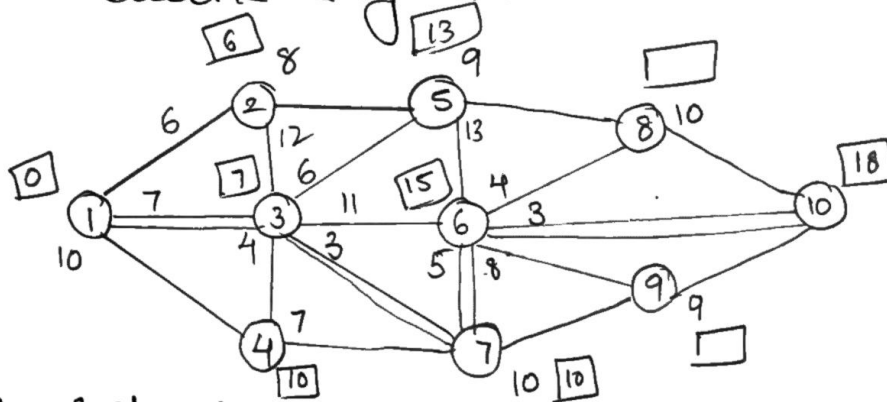
Step 4 :- Include the newly selected node in the "list A" of the next table say table-2

Step 5 :- Identify the nearest nodes to each node selected in the list A.

step 6: Calculate the cumulative distance from the new nodes to the selected nodes in the list A. Draw a square around the newly selected node.

step 7: Delete all columns (connecting lines) that are pointing towards newly selected nodes.

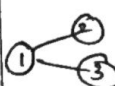
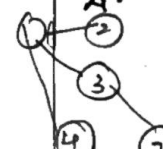
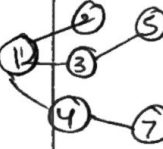
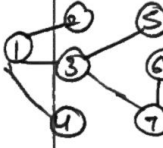
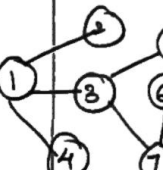
step 8: Repeat the procedure, to reach the last node in the network diagram.



By systematic method

1	2	3	4	5	6	7	8	9	10	
1-2 6	2-1 6	3-7 3	4-3 4	5-3 6	6-10 3	7-3 3	8-6 4	9-6 8	10-6 3	
1-3 7	2-5 8	3-4 4	4-7 7	5-2 8	6-8 4	7-6 5	8-5 9	9-10 9	10-8 9	
1-4 10	2-3 12	3-5 6	4-1 10	5-8 9	6-7 5	7-4 7	8-10 10	9-7 10	10-9 10	
		3-1 7		5-6 13	6-9 8	7-9 10				
		3-6 11			6-3 11					
		3-2 12			6-5 13					

Details of Selection of Nodes

Iteration no	List A (nodes selection)	Nearest Node(s)	Distance Calculation
1. ①	-	1	0
2. ①-②	1	2	6*
3. 	1, 2	3	7*
		4	10
4. 	1, 2	3	0+7 = 7
		4	6+12 = 18
		5	0+10 = 10 6+8 = 14
		6	0+10 = 10*
		7	6+8 = 14
		8	7+4 = 11
		9	7+6 = 13
5. 	1 x	4	7+6 = 13
		5	7+11 = 18
		6	7+3 = 10*
		7	7+3 = 10*
		8	6+8 = 14
		9	7+6 = 13*
		10	7+11 = 18
6. 	1 x	5	6+8 = 14
		6	7+6 = 13*
		7	7+11 = 18
		8	10+5 = 15
		9	10+10 = 20
		10	7+11 = 18
		11	13+13 = 26
7. 	1 x	6	13+9 = 22
		7	10+5 = 15*
		8	10+10 = 20
		9	13+9 = 22
		10	15+4 = 19
		11	15+3 = 18*
		12	15+8 = 23
		9	10+10 = 20

Select the city 1 the cumulative distance travel upto city 1 is ~~set~~ ^{set} to '0'. The selected City 1 is consider as the nearest node. Draw a square around ~~the~~ it '□'.
 Delete all arcs (connecting lines to the city 1)
 Such as (2-1, 3-1, 4-1) #

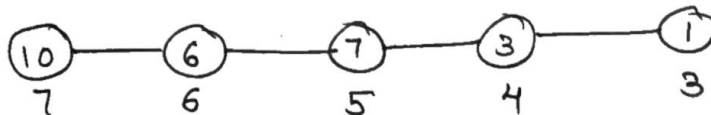
Iteration - 2

Select the city 2 (due to the less distance from the city 1)
 delete all arcs that are pointing towards city 2
 (1-2, 3-2 and 5-2). The nearest nodes are city 3
 and city 4

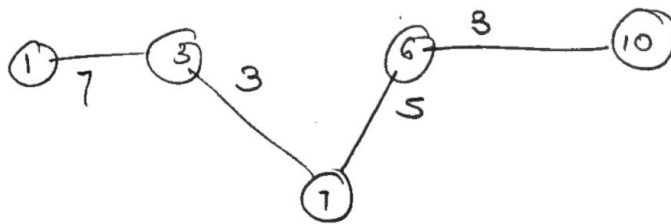
Conclusion :-

By backtracking method

the shortest path is



The duration on the total distance = 18 km



Dijkstra's Algorithm :-

This is another method to find the shortest distance (or) shortest path between any two nodes in a distance network. The arcs (connecting line - joining) may be direct or indirect. The procedural steps as follows.

Step 1 :-

Develop a distance matrix from the start node to all other nodes (If there is no direct arc from the start node to other nodes the distance may be assumed as " ∞ ")

Let select a start node be 'k'

Step 2 :-

find the smallest distance from the selected Node 'k' to all other nodes. Let the distance be 'x' and corresponding node be 'L'

(a) If it is the smallest distance, then "DARK". The arc (connecting line) between start node (k) to next neighbour node (L)

(b) select other nodes, that one at smallest distance

Step 3 :-

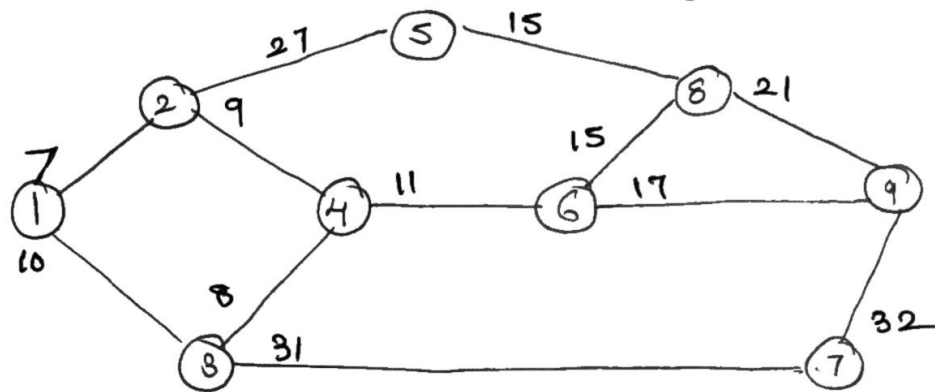
for each of unselected nodes (neighbours) of Node (L), find its new distance, by adding 'x' to its distance from the Node (L)

Transfer the last node [with distance] to the Column corresponding to the recently selected node (L)

Step 4 :- Trace the shortest path. fix the selected node of ~~last~~ last iteration as the last node of the shortest path.

Complete the construction of shortest path

Problem :- find the shortest - path Apply Dijkstra's Algorithm



Sol :- The distance matrix, summarizing the distances from the Node ① [K] to all other nodes is given below. This matrix represents the actual distances from the start Node 1 to all its neighbours nodes (direct connection) then all other nodes are assumed to be " ∞ ".

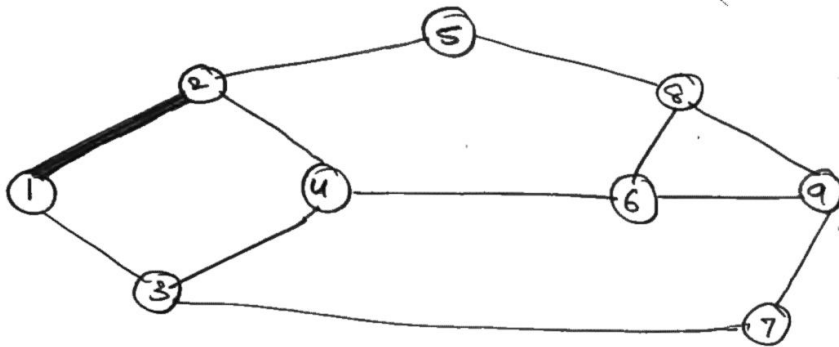
From the network diagram the smallest distances (K) and the corresponding node 2(L) since the node 2 is neighbour of the start Node ①, DARK (OR) THICK the arc (connecting line), (1-2) as shown.

Iteration - 1 Distance matrix :-

NODE	2	3	4	5	6	7	8	9
DISTANCE	7	10	∞	∞	∞	∞	∞	∞

↑

Node 2 is not the required destination go to the next nearest nodes

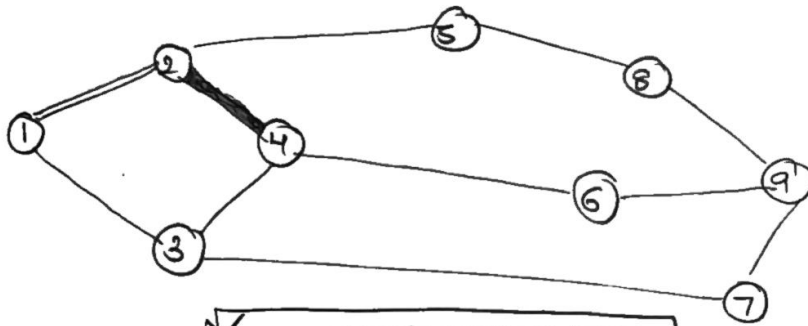


for the selected Node 2 [L]. the unselected nearest nodes are Node 5 and Node 4. The cumulative distances are

$$\textcircled{2} - \textcircled{4} = x + d_{2,4} = 7 + 9 = 16$$

$$\textcircled{2} - \textcircled{5} = x + d_{2,5} = 7 + 27 = 34$$

Remove the Node 2 and last Node comes to its position update the distance matrix as follows.

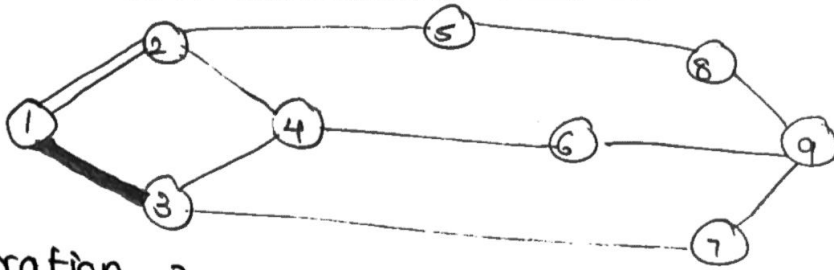


Node	1	2	3	4	5	6	7	8
distance	∞	∞	10	16	34	∞	∞	∞

Iteration 2

from the distance matrix, the smallest distance is (X) is 10, and the corresponding Node(L) is 3. since it is the neighbour of the start Node 1 "DARK (or) THICK" the line joining 1-3 in the network diagram as shown below.

Since node 3 is not the required destination update the distance matrix as shown below



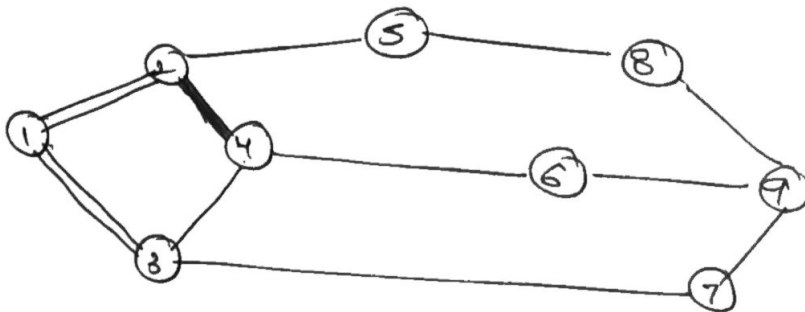
Iteration - 3 :-

Node	9	8	4	5	6	7
distance	∞	∞	16	34	∞	41

Distance matrix

From the matrix the shortest distance (x) is 16 and the corresponding Node is (4) since node 4 is not the neighbour of start node, the iteration continues find the cumulative distance to the Node (4)

The line joining of Node (2) & Node (4) as shown below.



Since node (4) is not the required destination continue the process

update the distance matrix is shown below

Delete Node (4) and the last node will come to the position.

Iteration - 4

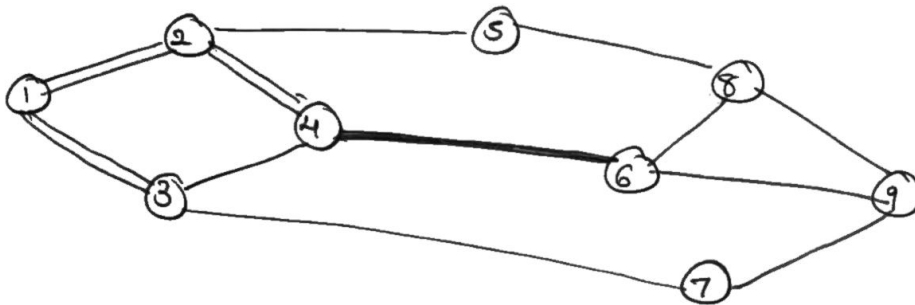
9	8	7	5	6
∞	∞	41	34	27

$$10 + 31 = 41$$

$$7 + 27 = 34$$

$$7 + 9 + 11 = 27$$

From the distance matrix, the shortest distance is (X) is 27 and corresponding Node is (6) as it is not the neighbour Node of start node. find the Cumulative distance. Now the DARK (or) THICK the line joining the Node (4) and Node (6) as shown



Since Node (6) is NOT the destination node Continue the process and update the distance matrix

Delete node (6)

Iteration - 5 :-

9	8	7	5
44	42	41	34

$$7 + 9 + 11 + 17 = 44$$

$$7 + 9 + 11 + 15 = 42$$

$$10 + 31 = 41$$

$$7 + 27 = 34$$

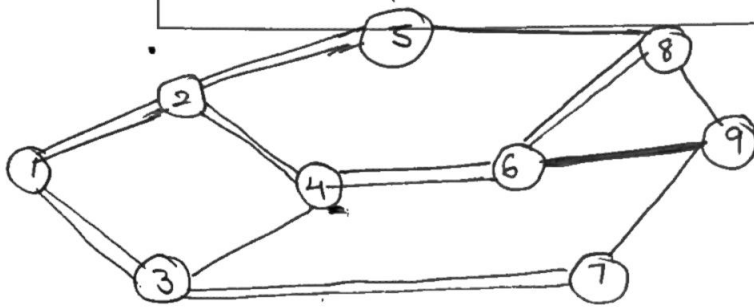
Iteration - 6

9	8	7
44	42	41

Iteration - 7

9	8
44	42

9
44



Hence from the network, the shortest path is
 1-2-4-6-9 the distance is 44 meters.

Floyd's Algorithm :-

This technique is applied to find the shortest path and corresponding distance from any source node to any destination node given in the network.

This method considers an initial distance matrix $[D^0]$ and initial precedence matrix $[P^0]$ as input this method algorithm performs "n" iteration [since the no. of nodes in the network is "n"] and moves to find (destination) final distance matrix $[D^n]$ and final precedence matrix $[P^n]$, with reference to shortest distance (from or between any two nodes); from the final distance matrix and corresponding paths from the final precedence matrix the path will be traced

The procedural steps as follows.

1. Develop initial distance matrix (D^0) and initial precedence matrix (P^0) from the given network.

2. The distance matrix and precedence matrix will have 'n' rows and n columns if there is no direct arc [connecting line joining], then the corresponding distance is assumed to be " ∞ ". The diagonal values are \in "zero".

In the precedence matrix the diagonal values are " " and all other entries in the i^{th} row are assumed as " i ", where $i = 1, 2, 3, 4 \dots n$

3. Consider the first node [say NODE ①]

4. find the values of distance matrix [D^k] for all entries, where i is not equal to j and apply the following relation:

$$D_{i,j}^k = \min \left[D_{i,j}^{k-1}, (D_{i,k}^{k-1} + D_{k,j}^{k-1}) \right]$$

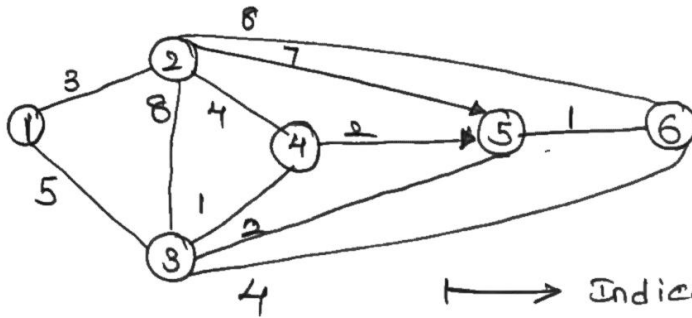
find the corresponding value in the precedence matrix [P^k] for all entries, where i is not equal to j and apply the following relation

$$P_{i,j}^k = P_{k,j}^{k-1} \left[\text{if } D_{i,j}^k \neq D_{i,j}^{k-1} \right]$$
$$= P_{i,j}^{k-1} \left[\text{if NOT} \right]$$

5. Continue the procedure until the last node is reached the path is the required shortest path from the source node to destination node

Problem :- (a) find the shortest path from source to destination.

(b) Also find shortest path from node 5 to node 1 and node 5 to node 2



→ Indicates one direction only.

Iteration - 0 :-

Distance matrix $[D^0]$

	1	2	3	4	5	6
1	0	3	5	∞	∞	∞
2	3	0	8	4	7	8
3	5	8	0	1	2	4
4	∞	4	1	0	2	∞
5	∞	∞	2	∞	0	1
6	∞	8	4	∞	1	0

Precedence matrix $[P^0]$

	1	2	3	4	5	6
1	0	1	1	1	1	1
2	2	0	2	2	2	2
3	3	3	0	3	3	3
4	4	4	4	0	4	4
5	5	5	5	5	0	5
6	6	6	6	6	6	0

Iteration - 1 :- node 1 is fixing

	1	2	3	4	5	6
1	0	3	5	∞	∞	∞
2	3	0	8	4	7	∞
3	5	8	0	1	2	4
4	∞	4	1	0	2	∞
5	∞	∞	2	∞	0	1
6	∞	8	4	∞	1	0

	1	2	3	4	5	6
1	-	1	1	1	1	1
2	2	-	2	2	2	2
3	3	3	-	3	3	3
4	4	4	4	-	4	4
5	5	5	5	5	-	5
6	6	6	6	6	6	-

Iteration - 2



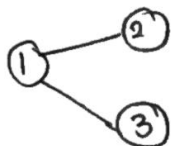
Backward

	1	2	3	4	5	6
1	0	3	5	7	10	11
2	3	0	8	4	7	8
3	5	8	0	1	2	4
4	7	4	1	0	2	12
5	8	8	2	8	0	1
6	11	8	4	12	1	0

	1	2	3	4	5	6
1	-	1	1	2	2	2
2	1	-	2	2	2	2
3	3	3	-	3	3	3
4	2	4	4	-	4	2
5	5	5	5	5	-	5
6	2	6	6	2	6	-

one new
so
8

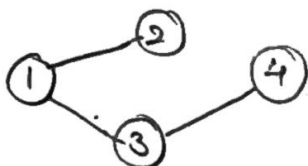
Iteration - 3



	1	2	3	4	5	6
1	0	3	5	6	7	9
2	3	0	8	4	7	8
3	5	8	0	1	2	4
4	6	4	1	0	2	5
5	7	10	2	3	0	1
6	9	8	4	5	1	0

	1	2	3	4	5	6
1	-	1	1	3	3	3
2	2	-	2	2	2	2
3	3	3	-	3	3	3
4	3	4	4	-	4	3
5	3	3	5	3	-	5
6	3	6	6	3	6	-

Iteration - 4



9, 11, 11

	1	2	3	4	5	6
1	0	3	5	6	7	9
2	3	0	5	4	6	8
3	5	5	0	1	2	4
4	6	4	1	0	2	5
5	7	7	2	3	0	1
6	9	8	4	5	1	0

	1	2	3	4	5	6
1	-	1	1	3	3	3
2	2	-	4	2	4	2
3	3	4	-	3	3	3
4	3	4	4	-	4	3
5	3	4	5	3	-	5
6	3	6	6	3	6	-

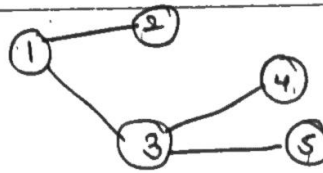
10, 9, 9, 7

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Iteration - 5

D^5

	1	2	3	4	5	6
1	0	3	5	6	7	8
2	3	0	5	4	6	7
3	5	5	0	1	2	3
4	6	4	1	0	2	3
5	7	7	2	3	0	1
6	8	8	4	4	1	0



P^5

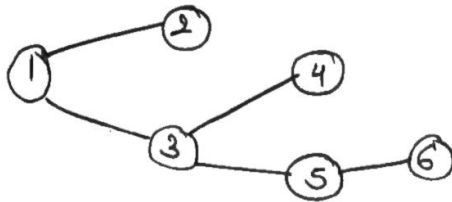
	1	2	3	4	5	6
1	-	1	1	3	3	5
2	2	-	4	2	4	5
3	3	4	-	3	3	5
4	3	4	4	-	4	5
5	3	4	5	3	-	5
6	3	6	5	3	6	-

- 1-2-3-5 = 13
- 1-2-5 = 10
- 1-2-4-5 = 9
- 1-3-4-5 = 8
- 1-3-5 = 7
- 1-3-2-5 = 20
- 1-3-2-4-5 = 20

Iteration - 6

	1	2	3	4	5	6
1	0	3	5	6	7	8
2	3	0	5	4	6	7
3	5	5	0	1	2	3
4	6	4	1	0	2	3
5	7	7	2	3	0	3
6	8	8	3	4	1	0

	1	2	3	4	5	6
1	-	1	1	3	3	5
2	2	-	4	2	4	5
3	3	4	-	3	3	3
4	3	4	4	-	4	5
5	3	4	5	3	-	5
6	3	6	5	3	6	-



Minimum Spanning tree :-

The objective of minimum spanning tree is to connect the nodes of a network such that the total length of the arc is the minimum. There are two techniques for minimum spanning tree they are

- a) prim's algorithm
- b) kruskal's algorithm

a) prim's algorithm :-

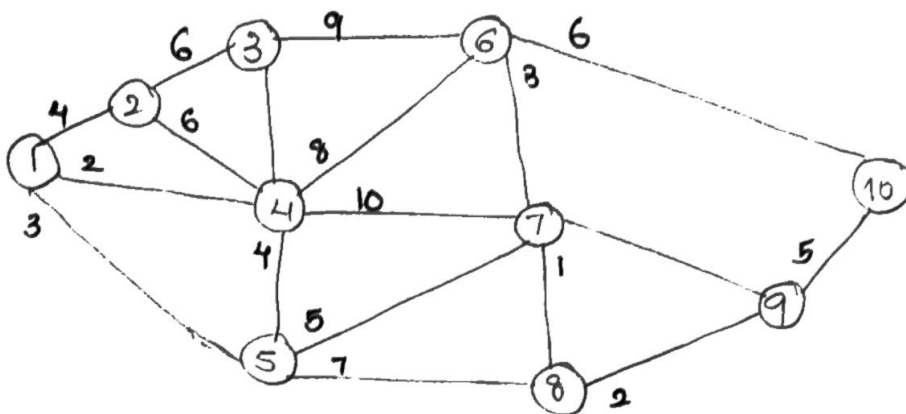
The objective of this algorithm is to find optimal solution for minimum spanning tree problem. The procedural steps as follows.

1. Represent the distance network in a matrix form
2. Let 'Q' is a null set, known as the set of selected row numbers of the matrix
3. select Row (1) and include it in 'Q'. Delete the Column (1) of the matrix
4. find the minimum of undeleted values, among the rows in Q, and select it by making a "SQUARE" (\square) around it. In case of tie, select randomly
5. Identify the corresponding column number (k) and then include row (k) in 'Q'
6. Delete the column 'k' of the matrix
7. Represent the arc (connecting lines) in the spanning tree, corresponding to the cells of the matrix with SQUARES, by thick lines

8. Find the sum of the values (marked with squares)
 It is the shortest path in minimum spanning tree.

problems :-

And the minimum spanning tree by



Iteration - 0

The Initial Distance matrix from Network as follows

	1	2	3	4	5	6	7	8	9	10
1	-	4	∞	2	3	∞	∞	∞	∞	∞
2	4	-	6	6	∞	∞	∞	∞	∞	∞
3	∞	6	-	5	∞	9	∞	∞	∞	∞
4	2	6	5	-	4	8	10	∞	∞	∞
5	3	∞	∞	4	-	∞	5	7	∞	∞
6	∞	∞	9	8	∞	-	3	∞	∞	6
7	∞	∞	∞	10	5	3	-	1	3	∞
8	∞	∞	∞	∞	7	∞	1	-	2	∞
9	∞	∞	∞	∞	∞	∞	3	2	-	5
10	∞	∞	∞	∞	∞	6	∞	∞	5	-

Iteration - 1

Let 'Q' be the NULL SET select node - ① include Node ① to 'Q' then $Q = \{1\}$.

The Select Row in 'Q' is indicated by "*".

The Distance matrix after deleting column - ① is as shown below

	2	3	4	5	6	7	8	9	10
* 1	4	∞	2	3	∞	∞	∞	∞	∞
2	-	6	6	∞	∞	∞	∞	∞	∞
3	6	-	5	∞	9	∞	∞	∞	∞
4	∞	5	-	4	8	10	∞	∞	∞
5	∞	∞	4	-	∞	5	7	∞	∞
6	∞	9	8	∞	-	3	∞	∞	6
7	∞	∞	10	5	3	-	1	3	∞
8	∞	∞	∞	7	∞	1	-	2	∞
9	∞	∞	∞	∞	∞	3	2	-	5
10	∞	∞	∞	∞	6	∞	∞	5	-

Iteration - 2

The minimum Cell value of undeleted columns in the 'Q' set is '2'. mark with it a square.

Now include node ④ to the 'Q' set

The selected Row in the 'Q' is indicated by the "*".

Delete the corresponding column. Now the distance matrix as shown below.



The distance matrix after deleting Row ④ is

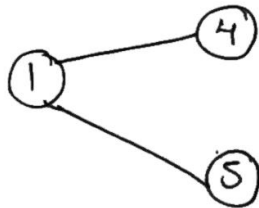
	2	3	5	6	7	8	9	10
* 1	4	∞	3	∞	∞	∞	∞	∞
2	-	6	∞	∞	∞	∞	∞	∞
3	6	-	∞	9	∞	∞	∞	∞
* 4	∞	5	4	8	10	∞	∞	∞
5	∞	∞	-	∞	5	7	∞	∞
6	∞	9	∞	-	3	∞	∞	6
7	∞	∞	5	3	-	1	3	∞
8	∞	∞	7	∞	1	-	2	∞
9	∞	∞	∞	∞	3	2	-	5
10	∞	∞	∞	6	∞	∞	5	-

Iteration - 3 :-

The minimum cell value in the selected rows in the 'Q' set is '3'

Now $Q = \{1, 4, 5\}$. Draw a square around that value. Delete the corresponding column 5 from the distance matrix. Put a '*' mark to the newly added node to the 'Q' set.

Now the distance matrix as below



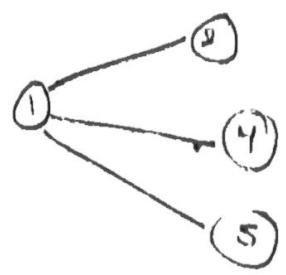
	2	3	6	7	8	9	10
* 1	4	8	8	8	8	8	8
2	1	6	8	8	8	8	8
3	6	1	9	8	8	8	8
* 4	6	5	8	10	8	8	8
* 5	8	8	8	5	7	8	8
6	8	9	1	3	8	8	6
7	8	8	8	1	1	8	8
8	8	8	8	1	1	2	8
9	8	8	8	3	2	1	5
10	8	8	6	8	8	5	1

Iteration-4

The minum cell value of selected rows in set $A = \{1, 4, 5\}$ is 4 in the column 2. Hence it is equal as shown above

Now delete column 2. Hence $A = \{1, 4, 5, 2\}$. The distance matrix is as shown below

	3	6	7	8	9	10
* 1	8	8	8	8	8	3
* 2	6	8	8	8	8	8
3	1	9	8	8	8	8
* 4	5	8	10	8	8	8
* 5	8	8	5	7	8	8
6	9	1	3	8	8	6
7	8	3	1	1	8	8
8	8	8	1	1	2	8
9	8	8	3	2	1	5
10	8	6	8	8	5	1



The minimum cell value in the selected row 4. Drop a square around the corresponding column 5

Iteration - 5

The minimum cell value in the distance matrix is 5 and the corresponding arc is (5) — (7).

Include node (7) into the ~~Q~~ ^Q set. now $Q = \{1, 2, 4, 5, 7\}$

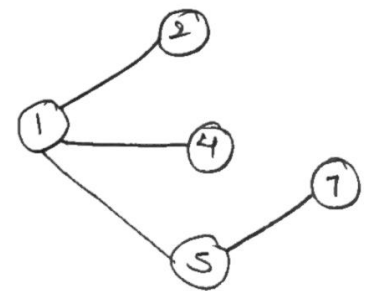
Drop a square around the selected minimum cell value

Drop a '*' mark indicate to the newly selected node (7).

Delete the corresponding column in the distance matrix

update the distance matrix. now iteration

	3	6	8	9	10
* 1	∞	∞	∞	∞	∞
* 2	6	∞	∞	∞	∞
3	-	9	∞	∞	∞
* 4	5	8	∞	∞	∞
* 5	∞	∞	7	∞	∞
6	9	-	∞	∞	6
* 7	∞	3	1	3	∞
8	∞	∞	-	2	∞
9	∞	∞	2	-	5
10	∞	6	∞	5	-



Iteration = 6

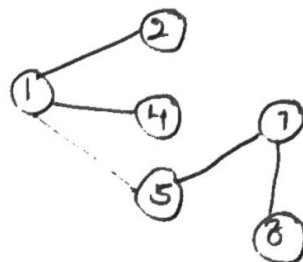
The minimum cell value distance matrix is 1

The corresponding arc is (7) — (8) . Include Node (8) to the

Q set . Now $Q = \{1, 2, 4, 5, 7, 8\}$. Drop a square around the selected minimum cell value . Drop '*' to newly selected node (8)

Delete the corresponding column . Now the updated distance matrix as follows.

		3	6	9	10
* 1		∞	∞	∞	∞
* 2		6	∞	∞	∞
3		-	9	∞	∞
* 4		5	8	∞	∞
* 5		∞	∞	∞	∞
6		9	-	∞	6
* 7		∞	3	3	∞
* 8		∞	∞	2	∞
9		∞	∞	-	5
10		∞	6	5	-



Iteration = 7

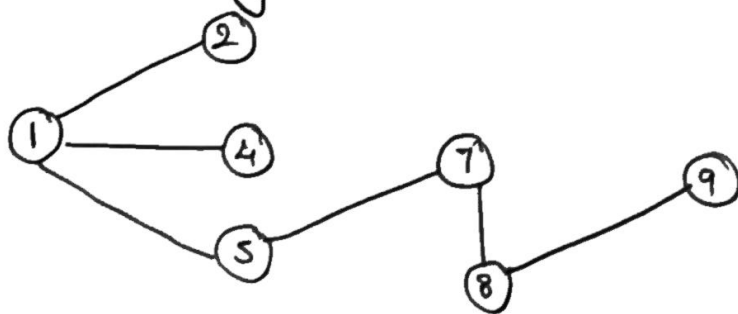
The minimum cell value in the distance matrix is '2'

The corresponding arc is (8) — (9) . Include node (9) to the Q set . Now $Q = \{1, 2, 4, 5, 7, 8, 9\}$.

Drop a square around the selected minimum cell value

Delete the corresponding column (9) . Now update the distance matrix

Draw '*' to newly selected node (9)



		3	6	10
* 1		∞	∞	∞
* 2		6	∞	∞
3		1	9	∞
* 4		5	8	∞
* 5		∞	∞	∞
6		9	1	∞
* 7		∞	3	∞
* 8		∞	∞	∞
* 9		∞	∞	5
10		∞	6	1

Iteration - 8

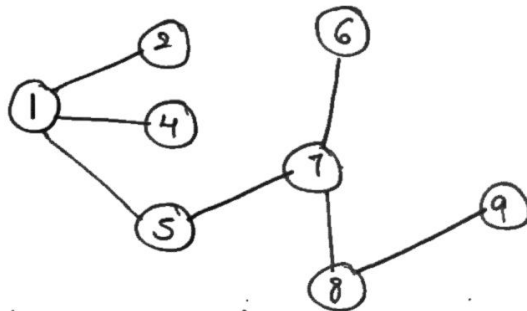
The minimum cell value in the distance matrix is '3'

The corresponding arc is (7) — (6)

Include node (6) to the Q set. Now $Q = \{1, 2, 4, 5, 6, 7, 8, 9\}$

indicate the newly selected node '*'. And delete corresponding column, update the distance matrix.

		3	10
* 1		∞	∞
* 2		6	∞
3		1	∞
* 4		5	∞
* 5		∞	∞
* 6		9	6
* 7		∞	∞
* 8		∞	∞
* 9		∞	5
10		∞	1

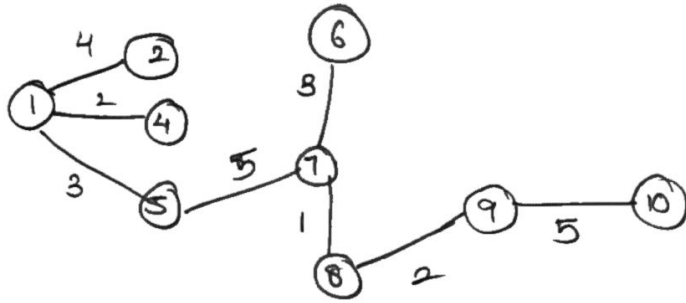


Iteration - 9 :-

The minimum cell value in the distance matrix is '5'.

The corresponding arc is (9) - (10). Include node (10) to the Q set. Now $Q = \{1, 2, 4, 5, 6, 7, 8, 9, 10\}$. Draw a square around the minimum cell value. Drop '*' to newly selected node (10). Delete the corresponding column in the distance matrix. update the distance matrix

	3
* 1	∞
* 2	6
3	-
* 4	5
* 5	∞
* 6	9
* 7	∞
* 8	∞
* 9	∞
* 10	∞



Iteration - 10 :-

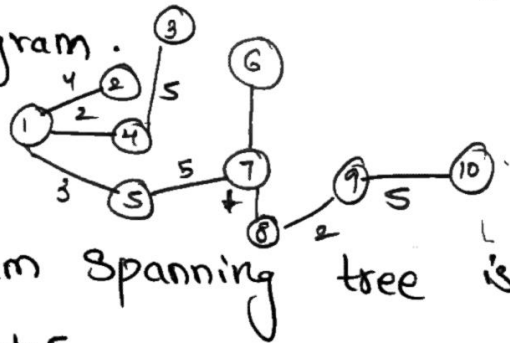
The minimum cell value in the distance matrix is '5'.

The corresponding arc is (4) - (3). Include node (3) to the Q set

The $Q = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Draw a square around the minimum cell value. Drop '*' to newly selected node (3).

Delete the corresponding column in Delete cheyathe local box kabati Delete rayacudhu. observe that all the nodes connected in network diagram.

Hence the iteration stops.



Conclusion :-

The total distance in the minimum spanning tree is

$$= 4 + 2 + 3 + 5 + 3 + 2 + 1 + 5 + 5$$

$$= 30$$

Kruskal's Algorithm :-

Consider the arc's [connecting line joining]. In case of undirected network develop a table, summarizing the distance of each arc (i, j) in the network diagram (where $i < j$)

the procedural steps as follows.

Step 1 :- Arrange the arc's in the given network diagram in the "Ascending order". This may be treated as 'm'

Step 2 :- Draw the network diagram only with the nodes (without connecting lines arc's).

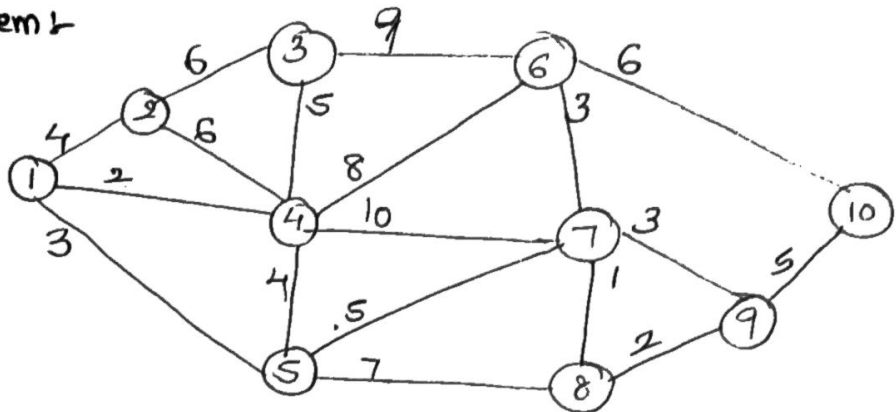
Step 3 :- select the smallest arc in the set 'm' (table) and ~~select~~ connect those nodes now delete the arc's from the selected list or m-set and continue.

Step 4 :- select all arc's successly and ~~add~~ the set 'm' is empty

Step 5 :- find the total distance in the minimum spanning tree Note :- avoid multiple connecting paths or routes

Note Connect one node one time only

problem 1



By Kruskal's algorithm to find the total minimum spanning

ARC	Distance		
1-2	4	6-10	6
1-4	2	7-4	10
1-5	3	7-5	5
2-1	4	7-8	1
2-3	6	7-9	3
2-4	6	8-5	7
3-2	6	8-7	1
3-4	5	8-9	2
3-6	9	9-7	3
4-1	2	9-8	2
4-2	6	9-10	5
4-3	5	10-6	6
4-5	4	10-9	5
4-6	8		
4-7	10		
5-1	3		
5-4	4		
5-7	5		
5-8	7		
6-3	9		
6-4	8		
6-7	3		

from the network diagram the distance matrix as follows,
 Arrange the Distance matrix in the ascending order

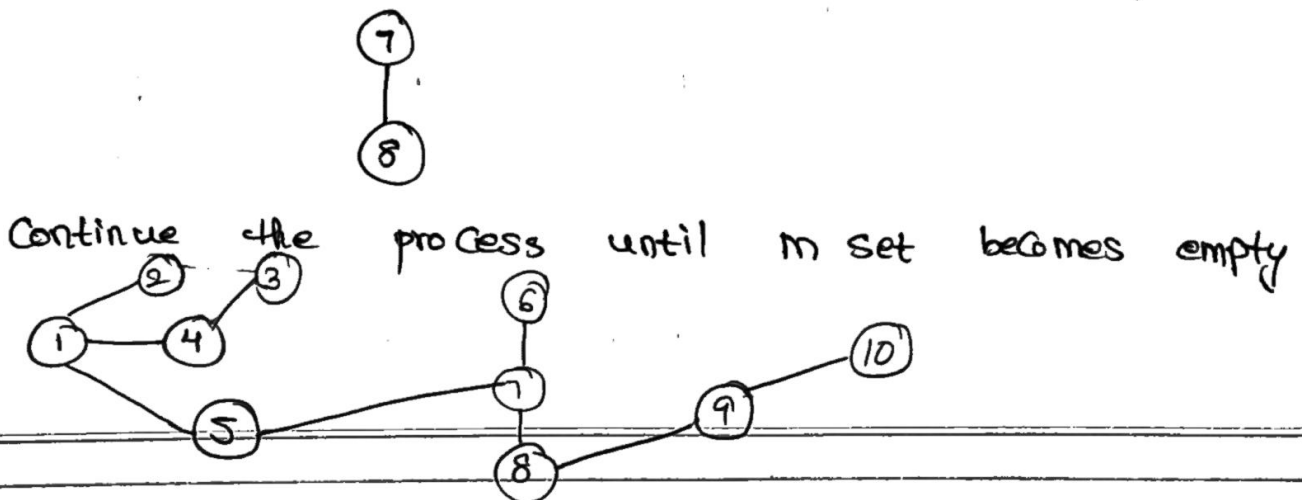
ARC	Distance		
7-8	1 ✓	5-7	5 ✓
1-4	2 ✓	9-10	5 ✓
8-9	2 ✓	2-3	6 x
1-5	3 ✓	2-4	6 x
6-7	3 ✓	6-10	6 x
x 7-9	3	5-8	7 x
1-2	4 ✓	4-6	8 x
x 4-5	4	3-6	9 x
8-4	5 ✓	4-7	10 x

cyclic

Now $m\text{-set} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

MINIMUM SPANNING TREE DIAGRAM

Select the smallest distance in the 'm' set and delete the same from the set



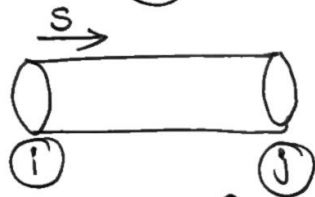
Conclusion The total distance in the minimum spanning tree = 30

Maximum flow problem :-

Let us consider an example of pipe network problem this is used to transfer fluid (in the liquid state such as oil, water, solvent and so on) from one location to another, the maximum flow of the fluid in each pipe segment is limited and based on factors such as diameter of the pipe, slope (inclination) of the pipe, obstacles, stoppage of flow, interruptions in the flow, clogs in the flow and soon. The flow of fluid in the pipe segment in the network diagram is broadly classified into the following two types.

1. Maximum permitted flow of fluid per unit time, from node (i) to node (j)
2. Maximum permitted flow of fluid per unit time, from node (j) to node (i)

Note :-



only one side flow (direction) is permitted, if zero is allotted or specify as ~~0~~ show in the pipe segment.

The purpose of this algorithm is to determine the quantity of flow from source to destination the solution to this problem can be derived by the following two methods

1. Linear programming model for the maximum flow
2. maximum flow ~~page~~ problem algorithm.

maximum flow problem Algorithm :-

The objective of this problem is to find maximum flow of fluid from source to destination.

The procedural steps as follows.

Step 1 :- from the given flow network diagram, develop a flow matrix (from-to capacity matrix)

A_{ij} is the Quantity of flow from node (i) to node (j) (where $i = 1, 2, 3, \dots, n$ and $j = 1, 2, 3, \dots, n$) and 'n' is the total number of nodes in the network. Set the iteration number ($k = 1$) and cumulative flow ($X = 0$)

Step 2 :- Select a path from the given source to destination (directly or passing through middle nodes, that have a quantity feasible for flow)

Step 3 :- find the minimum possible quantity of flow through various links in the selected path let this is as " Q_k ".

The new cumulative flow quantity $X = X + Q_k$

Step 4 :- Develop next flow matrix by the following rules.

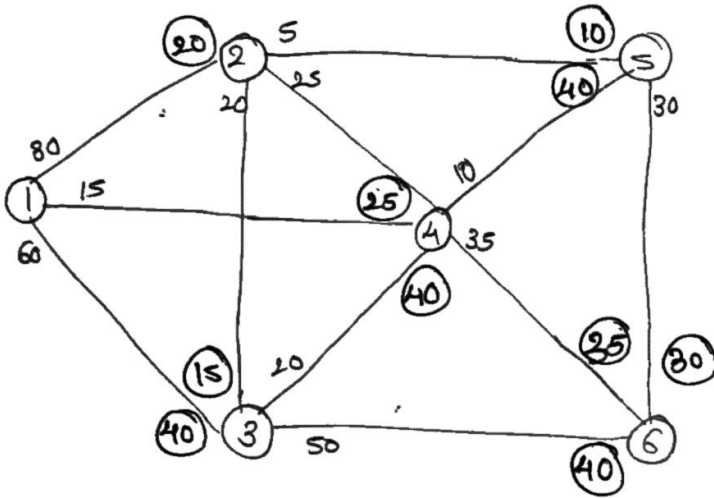
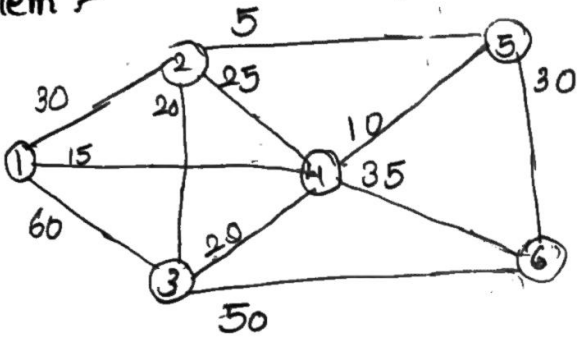
(a) subtract (Q_k) from all $A_{i,j}$ values, corresponding to all forward links of the path selected

(b) Add (Q_k) to all A_{ij} values corresponding to all "Backward links" of the path selected

Step 5 \rightarrow Now set $K \Rightarrow K+1$. go to the step 2

Step 6 \rightarrow Repeat the procedure until the destination node is reached.

problem \rightarrow



By the data the initial flow matrix is develop from the network diagram it is as follows

flow matrix

	1	2	3	4	5	6
1	-	30	60	15	-	-
2	20	-	20	25	5	-
3	40	15	-	20	-	50
4	25	25	40	-	10	35
5	-	10	-	40	-	30
6	-	-	40	35	30	-

Iteration - I

To stop the starting from the NODE ① the following path is selected

Initially, $x = 0$

The path is ① — ③ — ⑥

The quantity of flow $Q_1 = \min(60, 50) = 50$

Hence, the feasible flow quantity through this path is 50

Now assign "minus" sign to the path ① — ③ and ③ — ⑥

Assign positive sign to the cells 3-1 and 6-3 subtract Q_1 from the negatively assigned values

$$\begin{aligned} x &= x + Q_1 \\ &= 0 + 50 \\ &= 50 \end{aligned}$$

And add Q_1 to the positively assigned values

update the flow matrix

	1	2	3	4	5	6
1	-	30 ⁻	10	15	-	-
2	20 ⁺	-	20	25	5	-
3	50	15	-	20	-	-
4	25	25 ⁺	40	-	10	35
5	-	10	-	40	-	30
6	-	-	90	35 ⁺	30	-

Iteration - II Again start with the NODE ① the following path is selected path - 2 is ① — ② — ④ — ⑥

the minimum Quantity of flow in the line joining

$$Q_2 = \min(30, 25, 35) = 25$$

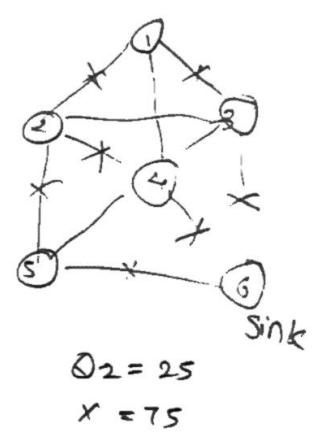
hence the feasible flow of Quantity in the path is 25

$$\begin{aligned} \text{Cumulative flow } X &= X + Q_2 \\ &= 50 + 25 \\ &= 75 \end{aligned}$$

Now assign "MINUS" sign to the values in path 1-2, 2-4, 4-6 and assign positive sign to the values in the paths 2-1, 4-2, 6-4

Subtract Q_2 from all negative signed values and add Q_2 to the all positively signed values
update the flow matrix

	1	2	3	4	5	6
1	-	5 ⁺	10	15 ⁻	-	-
2	25 ⁻	-	20	-	5	-
3	0	15	-	20	-	-
4	25 ⁺	50 ⁻	40	-	10	10 ⁻
5	-	10	-	40	-	30
6	-	-	90	60 ⁺	80	-



Iteration - III

Again start with the NODE ① the following path is selected for path - 3 is 1 - 4 - 6

from the flow matrix $Q_3 = (15, 10)$
 $= 10$

Hence this is feasible flow quantity in the path is 10

$$\begin{aligned} \text{Cumulative flow } X &= X + Q_3 \\ &= X + 10 \\ &= 75 + 10 \\ &= 85 \end{aligned}$$

Now assign minus sign to the values 1-4, 4-6

assign positive sign to the values 4-1, 6-4

Add Q_3 ~~to~~ all positively signed cell values

Subtract Q_3 from negative signed ^{matrix} cell values

Now update the flow matrix as follows.

Flow matrix

	1	2	3	4	5	6
1	-	5 ⁻	10	5	-	-
2	45 ⁺	-	20	-	5 ⁻	-
3	90	15	-	20	-	0
4	85	50	40	-	10	-
5	-	10 ⁺	-	40	-	30 ⁻
6	-	-	90	70	30 ⁺	-

Iteration 4

Again start from node ① the following path is selected

The path, ④ is 1 - 2 - 5 - 6

from the flow matrix $Q_4 = \min(5, 5, 30)$
 $= 5$

The feasible Quantity of flow by this path is 5

The Cumulative flow $X = X + Q_4$
 $= X + 5$
 $= 85 + 5$
 $= 90$

1-2 2-3 5-6

~~1-2 2-3 5-6~~

Now assign minus sign to the matrix/cells

Now assign positive sign to the cells/arc

2-1 5-2 6-5

Subtract Q_4 from all negatively signed cell and add

Q_4 to the all positively signed values

update the flow matrix as follows

flow matrix

	1	2	3	4	5	6
1	-	-	10 ⁻	5	-	-
2	50	-	20	-	-	-
3	90 ⁺	15	-	20 ⁻	-	0
4	35	50	40 ⁺	-	10	-
5	-	15	-	40 ⁺	-	25 ⁻
6	-	-	90	70	35 ⁺	-

← 1-3, 3-4, 4-5, 5-6
 5-1, 4-3, 5-6, 6-5

Iteration - 5

Again start with node ① and following path is selected

path-5 1 - 3 - 4 - 5 - 6 from flow matrix $Q_5 = \min\{10, 20, 10, 25\}$
 $Q_5 = 10$

The feasible Quantity ~~of~~ of flow through this path is 10

Cumulative flow $X = X + Q_5$
 $= 90 + 10$
 $= 100$

Now assign minus sign to the cells 1-3, 3-4, 4-5, 5-6
 assign positive sign to the cells 3-1, 4-3, 5-4, 6-5
 subtract Q_5 from all negatively signed ~~paths~~ arcs/cells and
 Add Q_5 to all positively signed cell values in flow values
 Update the flow matrix as follows.

flow matrix

	1	2	3	4	5	6
1	-	-	-	5	-	-
2	50	-	20	-	-	-
3	100	15	-	10	-	0
4	35	40	50	-	10	-
5	-	15	-	50	-	15
6	-	-	90	10	45	-

Now further feasible flows from node ① are not possible to the NODE - 6

subtract matrix elements of iteration-5 from the corresponding elements of flow matrix of given problem.
 write only positive values in the final resultant matrix.

The resultant matrix as follows.

Final flow matrix

	1	2	3	4	5	6
1	-	30	60	10	-	-
2	-	-	-	25	5	-
3	-	-	-	10	-	50
4	-	-	-	-	10	35
5	-	-	-	-	-	15
6	-	-	-	-	-	-

maximum flow from node ① to node ⑥ = 100 units