Unit - 5

Project Management & Scheduling

Definition :-

A product is defined as Combination of Inter related activities, they must be Executed in a Specified Completion the activities are Inter-related in a logical sequence. such that some activities Can't start until some other activities are Completed

An activity in a project is a job requiring time and utilization of resource for Completion The two activities known as C.P.M and P.E.R.T are developed by 1956-58 by walker (E.L DUPOINT AND LEMOURS COMPANY) and a Team of engineers (POLAR MISSIE PROGRAM of US NAVY). These Techniques are time oriented methods.

→ **Techniques of project management :-**

The following are the few techniques

1. Critical part method (CPM)
2. program evaluation and Review technique (P.E.R.T)

3. Minimum Spaning tree
4. Shortest route Algorithm
5. Maximum flow Algorithm
6. Maximum flow with minimum Cuts

→ Applications of project Management :-

1. Construction of a dam or Canal
2. Construction of an Appartment or multistorage building
3. Maintainance of Aerospace industry
4. maintainance of oil refinery
5. Space Shettle

→ Basic Steps in project Management :-

project management and scheduling Consists of the following main four steps

a) planning :-

This phase starts with spliting or dividing the total project into small units. They are further divided into Activities and analysed. The inter idependency and logical relations are established.

b) scheduling :-

The objective is to develop a time chart, showing or indicating start and finish times of each activity based on the inter-relationship. they must identify critical part activities that are vital. the non-critical activities will show / indicate slack or flow types times, such that they can be delayed. the resources required are man, machine, capital, material, time and others

c) Allocation of Resources :-

The resources of allocation and conflicting since the demand varies, allocation of resources must be systemised. This is highly depend on management decission.

d) Controlling :-

CPM. deals with application of principles of management to identify areas that are critical for completion time wise for progress report updation of network, better financial and technical control is required

→ Network diagram Representation :-

The network diagram indicates inter dependency precedence relationship, logical relation among activities of a project

a) Activity :-

Any Indivisual operation, that consume resources, having a starting and ending is known as Activity

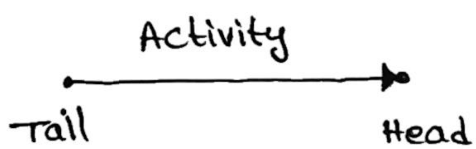
An arrow is used to represent an activity. The arrow head indicates the direction of progress in the project

1. predecessor activity :-
Activities that must be Completed before starting of another activity are known as predecessor activities

2. Successor activity :-
Activities that Cannot be started, until one or more other activities are completed, but immediately succeding them are known as Sucessor activities

3. Concurrent Activity :-
Activities that are occured simultaneously or at the same time are known as Concurrent activities

4. Dummy Activity :-
An activity that does not consume any resource, but is useful to represent technical dependency is known as dummy activity. They are represented by dotted lines



→ Event or Node :-
An event represents a point of time, Specifying the Completion and beggining of an activity. It is represented by the circle "O"
It is also known as "Node". they are classified into 3 types

a) Merge event :-

when more than one activity Come and enters an event it is known as merge event

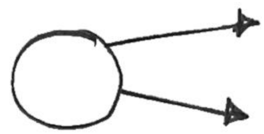
b) Burst event :-

when more than one activity leaves an event is known as burst event

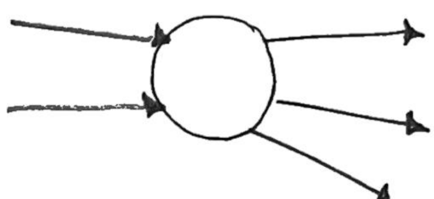
c) Merge and Burst event :-

An event may be merge and burst at the Same time is known as merge and berst event

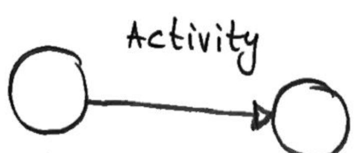
Merge event



Burst event



Merge & Burst event



Tail event Head event

Sequencing :-

Basel on interdependecy relation and logical sequence. The network diagram is Constructed placing or positioning activities on the above is known as network diagram

Rules for drawing the network diagram :-

The following are rules that are summeried for diagr Network diagram

1. Each activity is represented by only one arrow in the network diagram

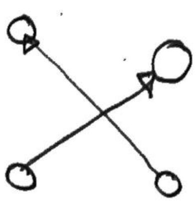
2. Two activities Can't be identify by the same end events

They must not be two activities have same starting event and ending event

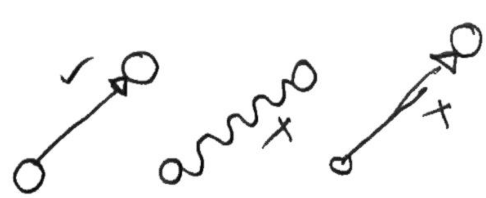
3. Dummy activities Can be utilized to establish the logical relation

4. The following are few rules

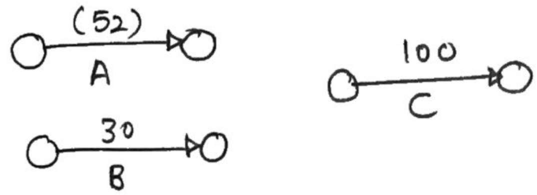
(i) Avoid arrows that cross each other



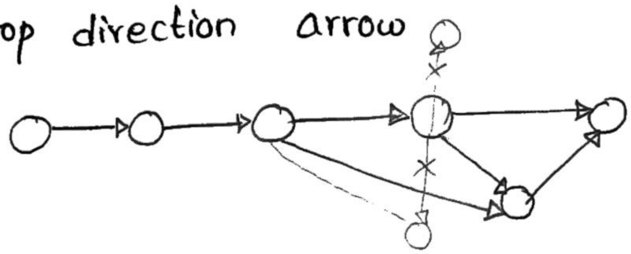
(ii) straight arrows are utilized



(iii) The length of arrow does not represent the duration of the activities



(iv) progression (or) propagation of arrows is from left to right and from top to bottom. Avoid bottom to top direction arrow



(v) Avoid mixing of two directions [: vertical & stright line]

(vi) Avoid dummy activities in the final network diagram

(vii) There should be only one start event and one end event in the network diagram

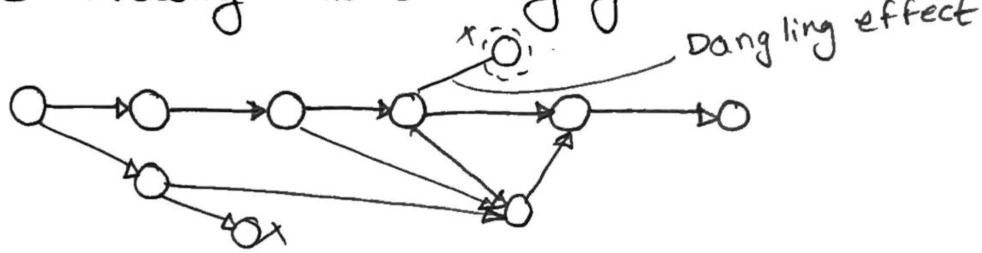
Common errors in the network diagram :-

there are three types of errors that are present frequently in the network diagram.

1. Dangling effect :-

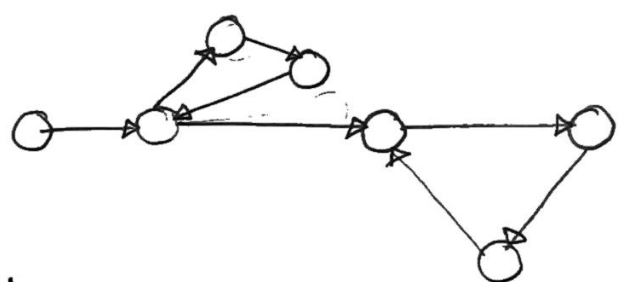
To dis Connect an activity before the Completion of all activities in the network diagram

The following shows Dangling effect



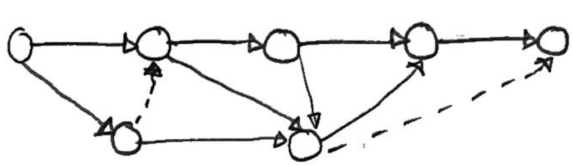
2. looping or Cycling error :-

The looping or cycling in the network diagram is an error it is shown below



3. Redundancy :-

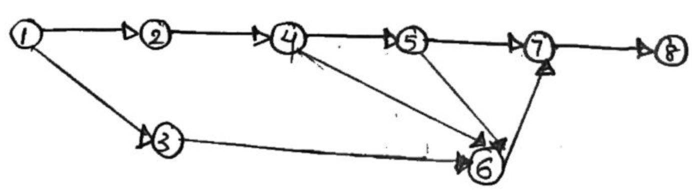
unnecessarily inserting dummy activities in the network diagram is known as Redundancy



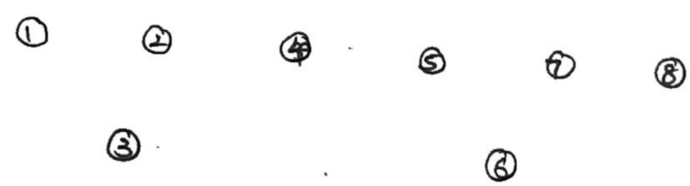
Fulkerson's Rule for Labeling the nodes (or) Numbering Nodes

It is also known as numbering the nodes or events In the network diagram all nodes must be properly numbered a standrad procedure developed by Fulkerson is generally utilized the procedural steps as follows

1. Identify the starting node that has arrows emerging from it number this event as unity (or) one
2. Delete all arrows emerging from the numbered node this will generate atleast one new start event
3. Number all the new start events has 2, 3, 4 - - - [In the ascending order]
4. Start the numbering of nodes from left to right and top to bottom.
5. Repeat the above procedure until the end event is reached



before



after delete

⇒ Critical path method (C.P.M)

The following terms are used

1. Critical Activity :-

* It is the activity that should "not be delay" ~~Cost/1~~
* If it is delay the entire project scheduled will be distrub.

2. Critical path :-

* The Sequence or chain or line joining of critical ~~path~~ activities of a project forms the Critical path
* It is the longest "duration path"

3. Critical path method (cpm)

* It is the technique of project management when duration of activities are known subtained and deterministic.
* It is represented by double line in the network diagram

for Critical Acitivity $|EST \sim LST| = |EFT \sim LFT| = 0$

Float for Critical Activity

$$Float = |EST \sim LST| = |EFT \sim LFT| = 0$$

⇒ Earliest Starting time [EST]

It is the earliest time that an activity Can stop from the begining of the project

Consider maximum value only

⇒ Earliest finish time :- [EFT]

It is the earliest time the activity Can finish from the begining of the project

⇒ **Latest Starting time :- [LST]**

It is the latest time that an activity can stop from the begining of the prodůct. But without delaying the Completion time of the project

⇒ **Latest finish time :- [LFT]**

It is the latest time, that an activity can be finished from the begining of the project without delaying the Completion of the project
"Consider Minimum value only"

⇒ **Time estimation and critical path :-**

In the Constructed network diagram time analysis is necessary for ~~playing~~ planing various activities of the Project

The object is to develop a planing schedule of the project.

The following factors are Consider for project Schedule.

1. Total Completion time for the project
2. Earliest time when an activity can stop
3. Latest time, when an activity can be started without

delaying the total project duration

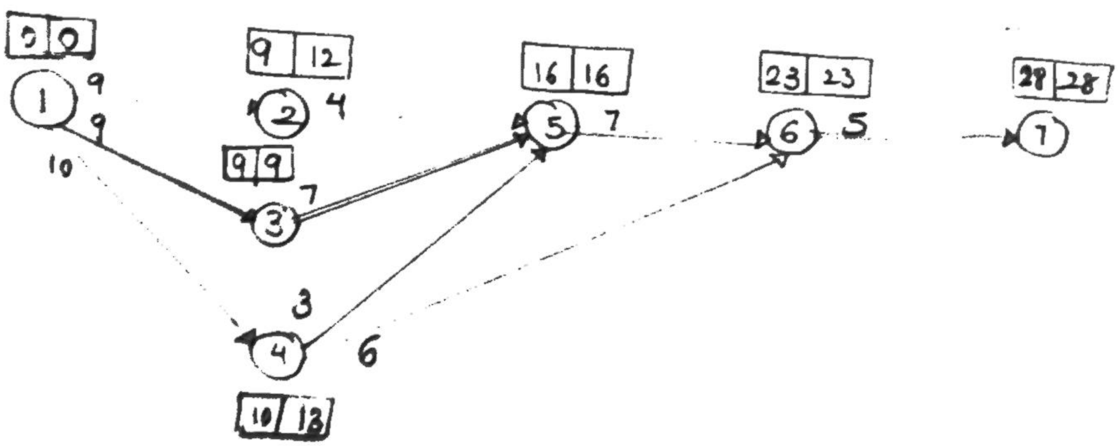
4. Float time for each activity

5. Identification of Critical activities and Critical path

problem :-

Draw the network diagram find Critical path ?

Activity	Duration	Start time EST	Start time LST	finish time EFT	finish time LFT	Float
1-2	9	0	3	9	12	
1-3	9	0	0	9	9	0
1-4	10	0	3	10	13	
2-5	4	9	12	13	16	
3-5	7	9	9	16	16	0
4-5	3	10	13	13	16	
5-6	7	16	16	23	23	0
4-6	6	10	17	16	23	
6-7	5	23	23	28	28	0



EST – Forword Recursive – Take maxim only

LST – Backword Recursion – Take minimum only

At Node 5

EST = Max { 9+4, 9+7, 10+3 }

= 16

At Node 6

EST = Max { 16+7, 10+6 }

= 23

At Node 4

LFT = Min { 23 − 6 , 16 − 3 }

= min { 17, 13 }

= 13

At Node 0

LFT = Min { 13 − 10, 9 − 9, 12 − 9 }

= 0

The Critical path of the project is

$1 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 7$

The maximum project duration = 28 days

Float :-

It is the amount of time available up to which the Completion of an activity Can be delay, without delaying the project Completion. It is the amount of time availabe to an activity that Can be utilize before, during and after the Performance of that activity

There are four types of floats available.

(i) Total float :- It is the absolute time difference b/w EST/ LST or EFT / LFT

Total flow is zero for Critical activity

Total float = | EST ~ LST | = | EFT ~ LFT | = 0

(ii) Free float :- It is the part of total flow, with in which the activity can be delay [manipulated], without affecting the float of succeding activities.

Free float = Total Float - Head event Slack

(iii) Independent float :- It is the part of total flow, with in which an activity can be delay [manipulated], without affecting the float of Preceding activities

Independent float = ~~Total~~ Free float - ~~Total~~ Tail Event Slack

Note :- If the negative value is obtained it can be taken as zero

(iv) Interfering float :-

Interfering float is that part of the total float which causes a reduction in the float of the successor activities

It is the difference between the latest finish time of the activity in question and the earliest starting time of the following activity or zero, whichever is larger

Interfering float = latest finish time of Activity under Consideration − Earliest starting time of the activity

[H ES]

Head event slack :- It is the absolute difference between latest event time and earliest event time at the head of an activitiy

Head event slack $= | E_i \sim L_i |$

$= | EST \sim LFT |$

Tail event slack (TES) :- It is the absolute difference between latest event time and earliest event time at the tail of an activity

Tail event slack $= | L_i \sim E_i |$

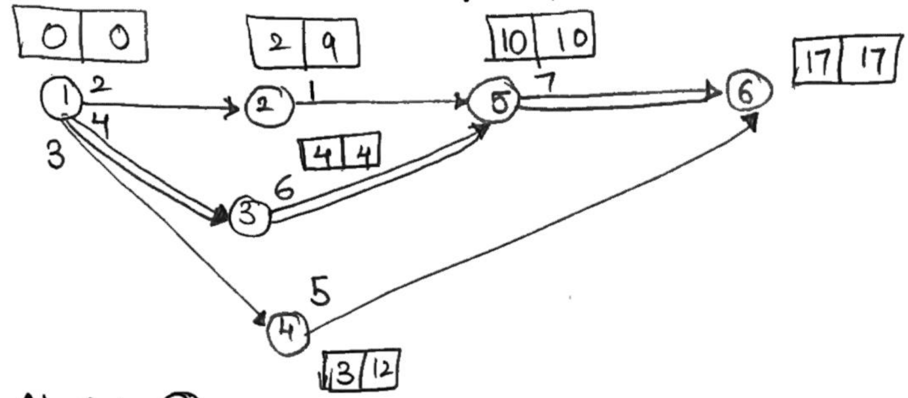
$= | LFT \sim EST |$

where E_i earliest expected occurance time of an event "i". L_i represents Latest allowed occurance time of an "event time".

Problem :-

4=6-2 5=3+2

Activity 1	Duration (days) 2	Start time		Finish time		Total float 7	Head event Slack	Free float	Tail event slack	Indepen-dent float
		EST 3	LST 4	EFT 5	LFT 6					
1-2	2	0	7	2	9	7	~~7~~ 0	0	0	0
1-3	4	0 —	0	4 —	4	0	0	0	0	0
1-4	3	0	9	3	12	9	9	0	0	0
2-5	1	2	9	3	10	7	0	7	7	0
3-5	6	4 —	4	10 —	10	0	0	0	0	0
4-6	5	3	12	8	17	9	0	9	9	0
5-6	7	10 —	10	17 —	17	0	0	0	0	0



At Node (5)

$EST = Max \{ 2+1, 4+6 \}$
$= 10$

At Node (6)

$EST = Max \{ 10+7 + 5+3 \}$
$= 17$

Come back return direction :-

$\left\{ \begin{matrix} 17-7 \\ 10-1 \\ 10-6 \\ 17-5 \end{matrix} \right\}$

At node (0)

$LFT = Min \{ 9-2, 4-4, 12-3 \}$
$= 0$

Activity	Head event Slack	Tail event slack
1-2	|2-9| = 7	10-0 = 0
1-3	|4-4| = 0	|0-0| = 0
1-4	|3-12| = 9	|0-0| = 0
2-5	|10-10| = 0	|2-9| = 7
3-5	|10-10| = 0	|4-4| = 0
4-6	|17-17| = 0	|3-12| = 9
5-6	|17-17| = 0	|10-10| = 0

Conclusion :-

the Critical path of the Netwoak project (1) → (3) → (5) → (6)

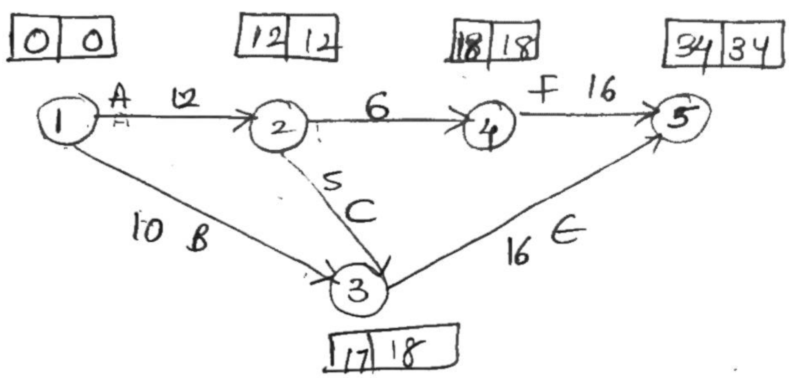
the (maximum) ~~Project~~ Duration of the project = 17 days

Problem :- 1

Activity :- A B C D E F

Predecessor :- _ _ A A B,C D

Duration :- 12 10 5 6 16 16



At node (3) $EST = \max\{0+10, 12+5\}$

$= 17$

at node (5) $EST = \max\{18+16, 17+16\}$

$= 34$

at node (2) $LFT = \min\{18-6, 18-5\}$

$= 12$

at node (0) $LFT = \min\{12-12, 18-10\}$

$= 0$

Activity	Activity	precedure	Duration	Start time EST	Start time LST	finish time EFT	finish time LFT	Total float	Head event slack	free float	Tail event slack	Independent
1-2	A	–	12	0	0	12	12	0	0	0	0	0
1-3	B	–	10	0	8	18	18	0	1	1	0	1
2-3	C	A	5	12	13	18	18	0	1	1	0	1
2-4	D	A	6	12	12	18	18	0	0	0	0	0
3-5	E	B, C	16	17	18	34	34	0	0	0	1	1
4-5	F	D	16	18	18	34	34	0	0	0	1	1

Activity	HES	Tail event slack
1-2	12-12=0	0-0=0
1-3	17-18 =1	0-0=0
2-3	17-18 = 1	12-12=0
2-4	18-18 = 0	12-12=0
3-5	34-34=0	17-18=
4-5	34-34=0	17-18=1

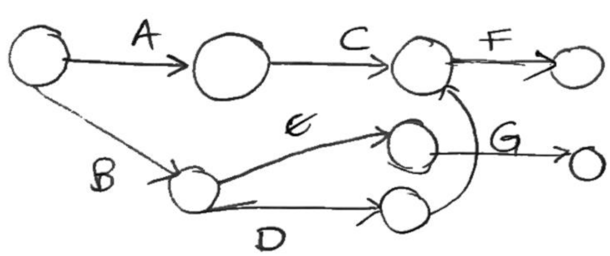
The critical path of the project is

$1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 5$

The minimum project Duration = 34 days

Problem :- 2

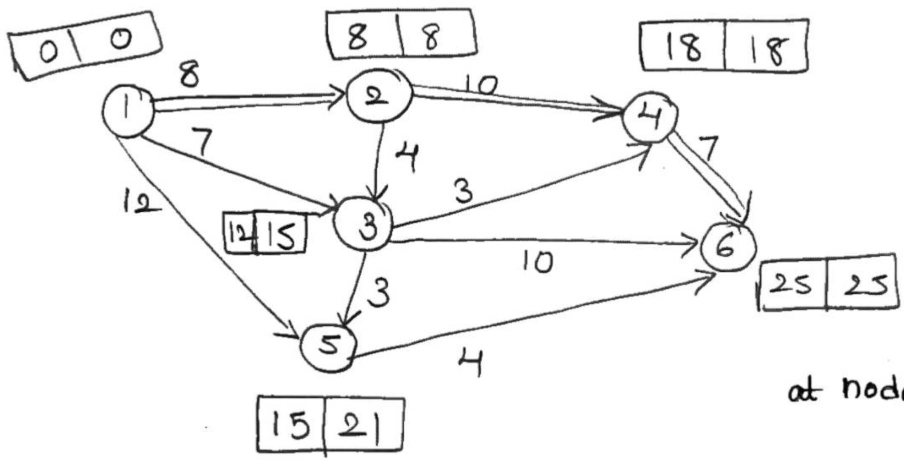
Activity :-	A	B	C	D	E	F	G
predecessor :-	–	–	A	B	B	C, D	F
Duration :-	6	5	4	4	4	6	6



MCA

Draw Network diagram find critical path and various time estimation

Activity	1–2	1–3	1–5	2–3	2–4	3–4	3–5	3–6	4–6	5–6
Duration	8	7	12	4	10	3	3	10	7	4



at node ⑥ $EST = \max\{18+7, 12+10, 15+4\}$

$= \{25, 22, 19\}$

$= 25$

at node ③

$EST = \max\{8+4, 0+7\}$

$= 12$

at node ④

$EST = \max\{8+10, 12+3\}$

$= 18$

at node ⑤

$EST = \max\{0+12, 12+13\}$

$= 15$

at node ③ $LFT = \min\{25-10, 18-3, 21-3\}$

$= \{15, 15, 18\}$

$= 15$

at node ② $LFT = \min\{18-10, 15-4\}$

$= 8$

at node ① $LFT = \min\{8-8, 15-7, 21-12\}$

$= 0$

Activity	Duration	start time		finish time		float
		EST	LST	EFT	LFT	
1	2	3	4	5	6	7
1–2	8	0	0	8	8	0
1–3	7	0	8	7	15	
1–5	12	0	9	12	21	
2–3	4	8	11	12	15	
2–4	10	8	8	18	18	0
3–4	3	12	15	15	18	
3–5	3	12	18	15	21	
3–6	10	12	15	22	25	
4–6	7	18	18	25	25	0
5–6	4	15	21	19	25	

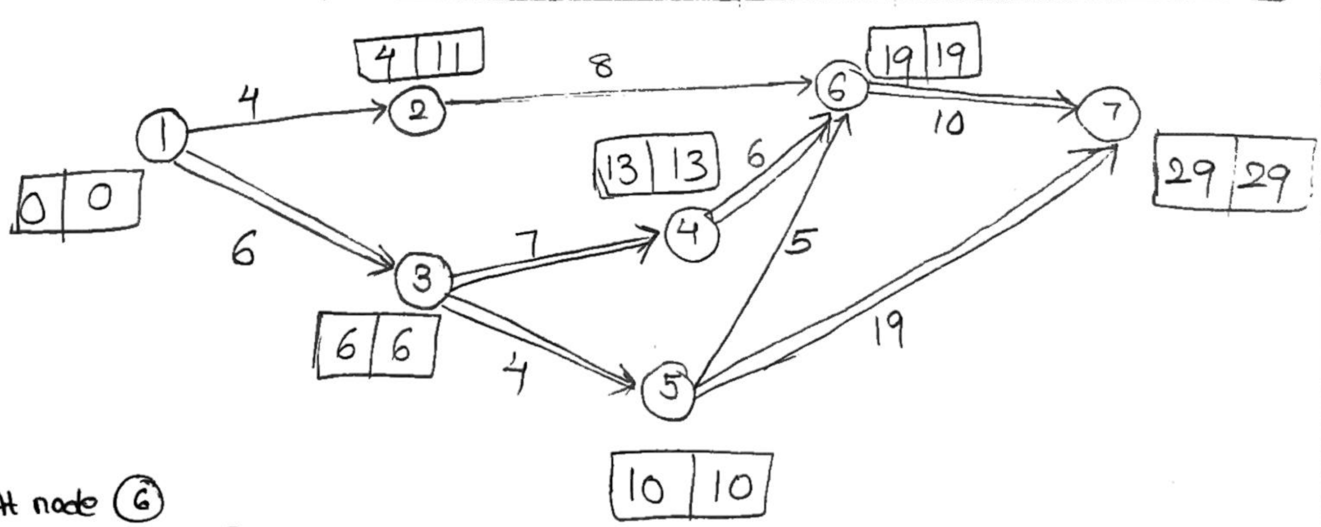
The critical path of the project is

1 → 2 → 4 → 6

∴ The maximum project duration = 25 days.

MBA
Oct/Nov 2022

Activity	Days Duration	Starting time		finishing time		float
		EST	LST	EFT	LFT	
1–2	4	0	~~7~~	4	11	
1–3	6	0	0	6	6	0
2–6	8	4	11	19	19	
3–4	7	6	6	13	13	0
3–5	4	6	6	10	10	0
4–6	6	13	13	19	19	0
5–6	5	10	14	15	19	
5–7	19	10	10	29	29	0
6–7	10	19	19	29	29	0



At node 6

$EST = \max\{10+5,\ 13+6,\ 4+8\}$

$= 19$

At node 7

$EST = \max\{10+19,\ 19+10\}$

$= 29$

at node 5 $LFT = \min\{19-5, 29-19\}$
$= 10$

at node 3 $LFT = \min\{13-7, 19-4\}$
$= 6$

at node 0 $LFT = \min\{6-6, 11-4\}$
$= 0$

∴ The critical path of the project is

$1 \rightarrow 3 \rightarrow 4 \rightarrow 6 \rightarrow 7$

$1 \rightarrow 3 \rightarrow 5 \rightarrow 7$

∴ The maximum project duration is $= 29$

programme Evaluation & Review Technique [P.E.R.T]

It is an event oriented technique

It is a propabilestic model

It considers uncertainities involved in the time estimation of an activity.

It consits of 3 Time - Estimations such as optimistic time, pessimistic time and most likely time.

It helps the project manager to schedule and Co-ordinate various activities such that the project can be Completed in a schedule time.

procedural steps Invlove :-

1. Identification of Activities of the Project
2. estimation of activity time.
3. follow or observe interdependency relation
4. Draw the network diagram
5. obtain the data by using network model

Differences between CPM & P.E.R.T

Point	Critical path method (C.P.M)	program Evaluation & Review Technique (P.E.R.T)
1. Orientation	Activity oriented	Event oriented
2. Feature of the model	* Deterministic model * uncertainties are not involve in the time estimation	* probabilistic model * Time estimations are involved.
3. purpose	* To balance between project time and project Cost * Focus will be on time and cost	* To balance between project ~~time schedule~~ time only * focuses on time only
4. Application	* utilized or applied short and repeatative project only * prior exprience is require	* Applied for long duration and non repeatitive projects * Applied for research and development projects

Applications of PERT :-

* It is Applied in facility planning (plan lay-out planning)
* Facility Cost and Control
* Application of principles of management
* Applied as project management schedule
* progress report or the status of the project

Terminology of P.E.R.T :-

uncertainities are incorporated (involve) in the P.E.R.T by assuming the activity times obey "beta distribution" [probability] this helps to calculate expected activity time and standard deviation

Optimistic Time (t_o) :- It is the minimum time require for completion of an activity, "under Ideal Conditions"

Most likely time (t_m) :- It is the most "probable time of normal time" for an activity completion time

pessimistic time (t_p) :- It is the maximum time for completion of an activity, Considering Delays, obstacles and stoppages

Expected time (t_e) :- It is the "average time" for completion for an activity in a project It is repeated for more number of times

It is given by the following relation :-

$$\boxed{t_e = \left[\frac{t_o + 4 \cdot t_m + t_p}{6}\right]}$$

project variance :- (σ^2) :-

It is the variance of critical path, that is normally distributed. It is the sum of variances of critical activities. It is applied to find the probability of completion of the project, In a given schedule time.

project variance = sum of variances of critical activities

$$\boxed{\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2}$$

Slack :-

It is the obsolute difference b/w $|E_i \sim L_i|$.

It is the amount of free time or additional time avaliable to the activity for completion without delaying the completion of the project

Slack ~~for~~ ~~Critical~~ of activity = $|E_i \sim L_i|$

Slack is zero for critical activity

Procedural steps for solution :- [PERT]

the procedural steps as follows.

1. Determine the expected time and various for every activity.
2. Draw the network diagram.
3. find the critical path
4. find the total expected duration of the project
5. It is also known as the mean duration of project completion
6. It is represented by the symbol "$(\bar{X})$"
7. Find the standrad deviation of the project.

$$\sigma = \sqrt{\text{sum of variances of critical activities}}$$

8. Calculate the probability of completion of the project
9. Calculate the value of Z.

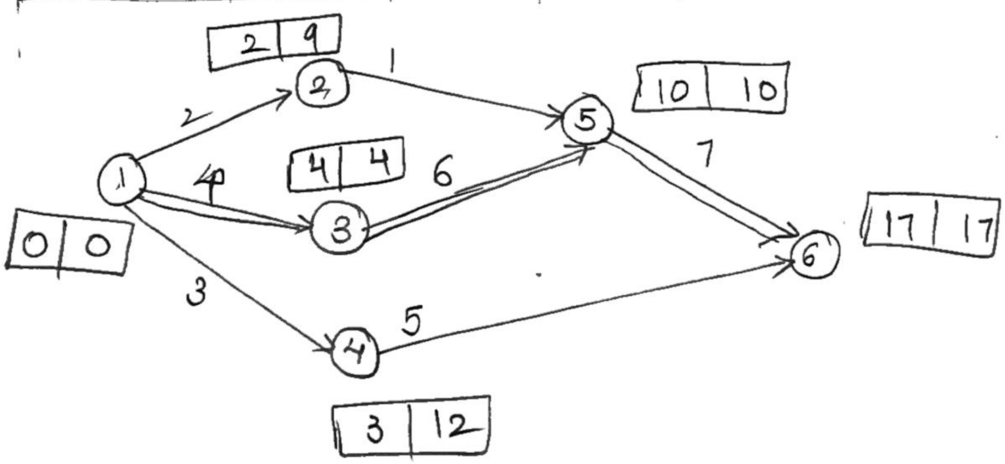
$$Z = \frac{X - \bar{X}}{\sigma}$$

where "X" represents calculated value of (given value) of projection completion.

10. find the value of z from the normal distribution table. find out the expected time of Completion time of project at a given probability

problem :-

Activity	Estimation of time			Expected Time t_e	variance σ^2
	t_o	t_m	t_p		
1-2	1	1	7	2	1
1-3	1	4	7	4	1
1-4	2	2	8	3	1
2-5	1	1	1	1	0
3-5	2	5	14	6	4
4-6	2	5	8	5	1
5-6	3	6	15	7	4



E_i	L_i

$$t_e = \frac{t_o + 4t_m + P}{6}$$

$$\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$$

At node 5

$E_i = \max\{2+1, 4+6\} = 10$

At node 6

$E_i = \max\{10+7, 5+3\} = 17$

At node 1

$L_i = \min\{9-2, 4-4, 12-3\}$

$= 0$

the Critical path for duration = 17 days

Total project variance = 1+4+4 = 9

Standard deviation $\sigma^2 = \sqrt{9} = 3$

By data

$x = 14$ days, Expected data $(\bar{x}) = 17$ days

$\sigma = 9$

The probability of completion in 14 days $(Z) = \frac{x - \bar{x}}{\sigma}$

$= \frac{14-17}{3}$

$Z = -1$

From the normal distribution ~~data~~ tables

$P(x \leq 14) = 0.1583 = 15.83\%$

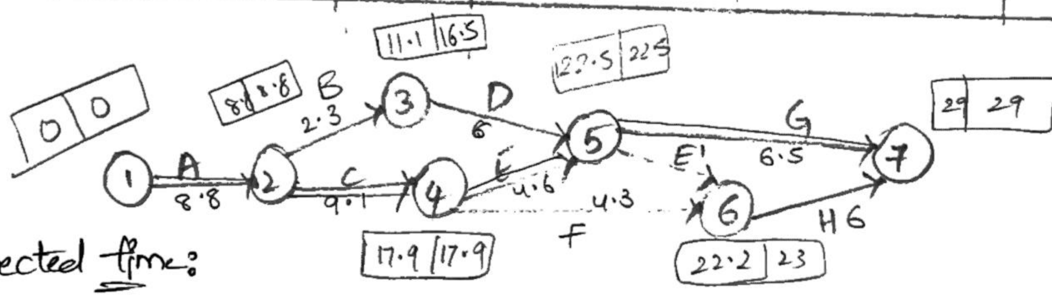
The probability of completion of the project in 14 days equal to 15.83%.

Calculation of slack:

Event	Ei	Li	slack
1	0	0	0
2	2	9	7
3	4	4	0
4	3	12	9
5	10	10	0
6	17	17	0

problem : MBA - Supplementary - MAY 2022 (CA)

Activity	precedence	Estimation of time			Expected time(t_e)	Variance (σ^2)
		t_o	t_m	t_p		
A (1-2)	–	2	9	15	8.8	4.4
B (2-3)	A	1	2	5	2.3	0.3
C (2-4)	A	7	8	16	9.1	2.2
D (3-5)	B	2	5	14	6	4
E (4-5)	C	2	3	14	4.6	4
F (4-6)	C	1	4	9	4.3	1.6
G (5-7)	D,E	6	7	5	6.5	001
H (6-7)	F,E	2	6	10	6	1.5



Expected time:

(i) $t_e = \left[\dfrac{t_o+4t_m+t_p}{6}\right] = \left[\dfrac{2+4(9)+15}{6}\right] = \left[\dfrac{2+36+15}{6}\right] = \dfrac{53}{6}.$

(ii) $t_e = \dfrac{1+4(2)+5}{6} = \dfrac{1+8+5}{6} = \dfrac{14}{6} = 2.3$

(iii) $t_e = \dfrac{7+4(8)+16}{6} = \dfrac{7+32+16}{6} = \dfrac{55}{6} = 9.1$

(iv) $t_e = \dfrac{2+4(5)+14}{6} = \dfrac{2+20+14}{6} = \dfrac{36}{6} = 6$

(v) $t_e = \dfrac{2+4(3)+14}{6} = \dfrac{2+12+14}{6} = \dfrac{28}{6} = 4.6$

(vi) $t_e = \dfrac{1+4(4)+9}{6} = \dfrac{1+16+9}{6} = \dfrac{26}{6} = 4.3$

(vii) $t_e = \dfrac{6+4(7)+5}{6} = \dfrac{6+28+5}{6} = \dfrac{39}{6} = 6.5$

(viii) $t_e = \dfrac{2+4(6)+10}{6} = \dfrac{2+24+10}{6} = \dfrac{36}{6} = 6$

Variance :- $(\sigma^2) = \left[\frac{t_p - t_o}{6}\right]^2$

(i) $\left(\frac{15-2}{6}\right)^2 = \left(\frac{13}{6}\right)^2 = (2.1)^2 = 44$

(ii) $\left(\frac{5-1}{6}\right)^2 = \left(\frac{\cancel{4}^2}{\cancel{6}_3}\right)^2 = \left(\frac{2}{3}\right)^2 = (0.6)^2 = 0.3$

(iii) $\left[\frac{16-7}{6}\right]^2 = \left[\frac{\cancel{9}^3}{\cancel{6}_2}\right]^2 = (15)^2 = 2.25$

(iv) $\left[\frac{14-2}{6}\right]^2 = \left(\frac{12}{6}\right)^2 = 2^2 = 4$

(v) $\left[\frac{14-4}{6}\right]^2 = \left(\frac{12}{6}\right)^2 = 2^2 = 4$

(vi) $= \left[\frac{9-1}{6}\right]^2 = \left(\frac{\cancel{8}^4}{\cancel{6}_3}\right)^2 = (1.3)^2$
$= 1.6$

(vii) $= \left[\frac{5-6}{6}\right]^2 = \left[\frac{-1}{6}\right]^2$
$= (-0.1)^2$
$= 0.01$

(viii) $\left[\frac{10-2}{6}\right]^2 = \left(\frac{\cancel{8}^4}{\cancel{6}_3}\right)^2 = (1.3)^2$
$= 1.6$

$\therefore$ The critical path is

$$1 \xrightarrow{A} 2 \xrightarrow{C} 4 \xrightarrow{E} 5 \xrightarrow{G} 7 \quad //$$

The project variance $= 4.4 + 2.2 + 4 + 0.01$
$= 10.61$

Standard deviation of the project $(\sigma) = \sqrt{10.61}$
$= 3.17$

The probability of Completion of the project in 25days

$$z = \frac{x - \bar{x}}{\sigma} = \frac{25-29}{3.1} = \frac{-4.0}{3.1} = 1.29$$

By data $x = 35.0$ days $\bar{x} = 29.0$ days $\sigma = 3.17$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{35-29}{3.17} = \frac{6.0}{3.17} = 1.89$$

from normal distribution tables the value is 0.4706

$P(X \leq 35) = 0.5 + 0.4706 \Rightarrow 0.9706 \Rightarrow 97.06\%$

+ value $-1.29 = 0.4015$
$\Rightarrow 0.5 - 0.4015$
$\Rightarrow 0.085$
$\Rightarrow 8.5\%$

The probability Completion of the project is 85% & probability of Completion of project in 35 days

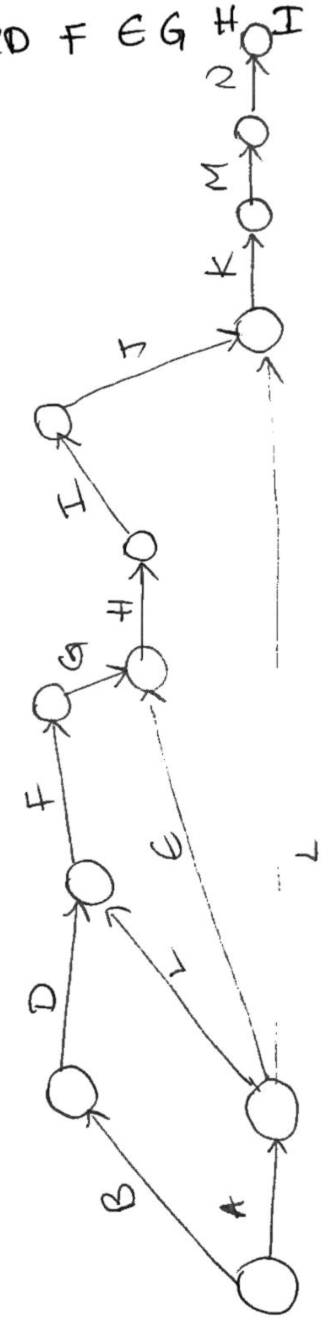
MBA – Regular oct / NOV 2022

Draw the Network diagram

Activity :- A B C D E F G H I J K L M N

precedence :- – – A B A CD F EG H I JL A K M

Activity	precedence
A	–
B	–
C	A
D	B
E	A
F	C, D
G	F
H	E, G
I	H
J	~~B~~ I
K	J, L
L	A
M	K
N	M

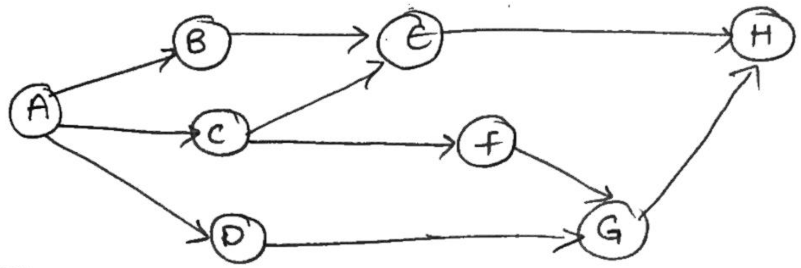


MBA :-

Supplimentary – may 2022

Describe the assumptions of depicting Network diagram

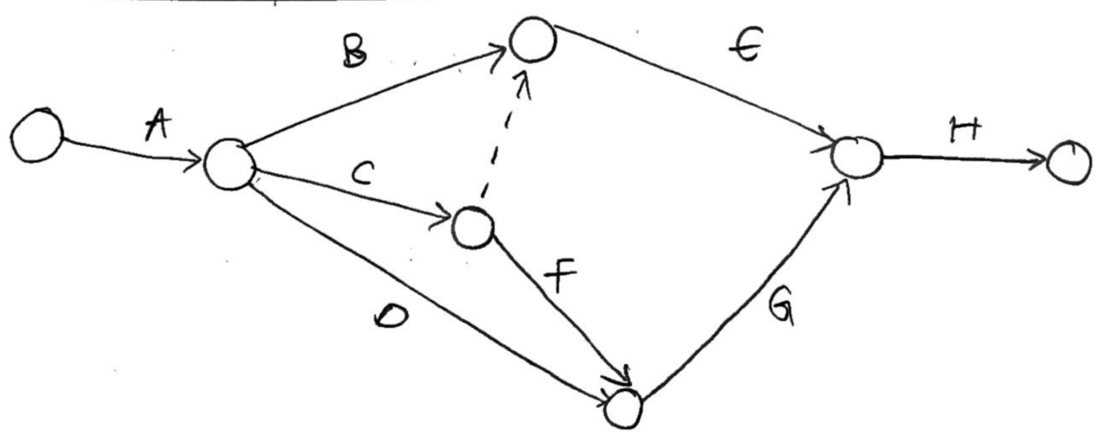
Convert below AON Network to AOA Network



AON

Activity	precedence
A	—
B	A
C	A
D	A
E	B, C
F	C
G	D, F
H	E, G

AOA



Network Construction

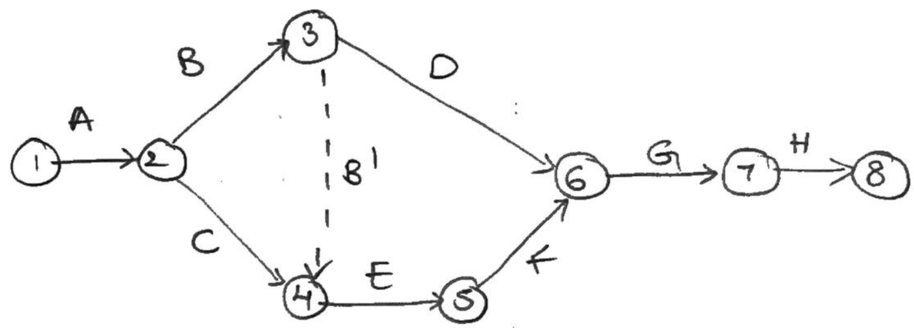
An activity A is Completed before standing another activity B we write A < B

A is preceding 'B' (or) Activity 'B' is Succeding

A < B, C , B < D, E ; C < E ; E < F ; D, F < G ; G < H

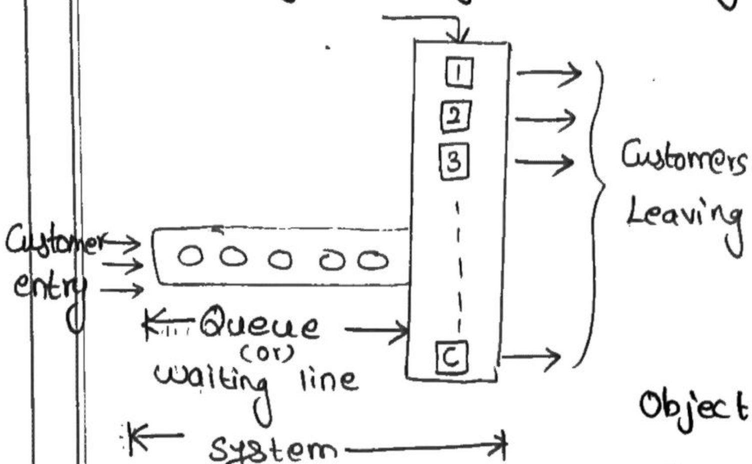
Sol

Activity	precedence
A	—
B	A
C	A
D	B
E	B, C
F	E
G	D, F
H	G



Activity	predecessor	Duration
A (1-2)	—	10
B (2-3)	A	20
B' (3-4)	—	Dummy
C (2-4)	A	30
D (3-2)	B	40
E (4-5)	B, C	50
F (5-6)	E	60
G (6-7)	D, F	70
H (7-8)	G	80

Queueing Theory (or) waiting line Theorey :-



Queuing model is suitable to represent or service oriented problem, where customers arrived ranbom to receive some service, the service is also a random variable

Objective :- The objective is to find the optimum service rote and number of servers, so that average time spent in the queuing system and the Cost of ~~syst~~ service or to weak minimize.

Terminology (or) terms associated :-

1. Customer :- It is the arriving unit or element or item or product that require some service
the customer may be person, machine, vechical, parts and so. on ...

2. Queue (or) waiting line :-
The number of customers waiting for the service will form a line or Queue it doesn't include that customer getting service

3. service channel :-
The process or system that delivers service to the customer is known as service channel. It may be single or multi-channel

The no. of service channels is denoted by the Symbol 'C'.

Characteristics of Queuing system :-

The following will specified the main characteristics of Queuing theory

(a) Input (or) Arrival (Inter-Arrival) Distribution :-

It represents the pattern in which the number of customers arrived at the System.
The arrival may also be represented by "Inter-Arrival time"
It is the time period between two successive arrivals.
The number of customers arriving per unit of time is known as " Arrival Rate".
It is described by POISSON distribution

(b) Service of (Departure) Distribution :-

It represents the pattern in which the number of Customers lead the System the departure may also be represented by service time [Inter departure]. It is the time period b/w two successive services the number of customers served per unit of time is known as "Service Rate".

It is described by exponential distribution.

(c) Service Channel :-

The Queueing model is known as "Single service model", when the system has only one server. In a "multi server model", there are a number of parallel channels with one server.

(d) Service Discipline :-

The service Discipline or order of service is the rule by which customers are selected from the Queue for service they are First Come First service (FCFS), Last Come Last service (LCLS) Random order (Ro)

The most Common descipline is FCFS.

~~The most common.~~

e) Maximum Number of Customers allowed in the System

The maximum Number of customers in the system may be either finite or infinite

f) Calling source (or) population :-

The arrival pattern of Customers depends upon the source that generates them the population Can be finite or infinite.

g) List of variables (or) used in the queuing system :-

Let 'n' - no of customers in the system [waiting line + under service]. At the time "t"

'C' - The no of Servers in the System

Pn (t) - probability of having 'n' customers in the System at the time 't'

Pn - Study state probability of having exactly 'n' Customers in the system.

Po - probability of having Zero Customers in the System

Lq - average number of customers waiting in the queue

Ls - average number of customers in the system [waiting + Service]

wq - average waiting time of Customers in the queue

ws - average no. of Customers in the System [waiting + Service]

λ – arrival way of customeas

μ – Service rate of the serveq

φ (or) ϕ – utilization factoa of the server. It is also known as traffic Intensity System business

M – Markovian or (poisson) distribution for arrivals
Exponential distribution foa Service

N – maximum numbeq of customeas permited in the system
It also denotes the size of Calling source of Customeas

GD – It represents General Serivce Descipline
FCFS, LCLS, RO (random order)

Basic Queueing model :-

The basic queueing model Can be Classified into 6 Categories
It is denoted by "Kendal Notation"

$(P/Q/R) : (X/Y/Z)$

wheae, P – arrival rate distribution
Q – service rate distribution
R – no. of servers
X – service discipline
Y – maximum no of Customers permited in system
Z – size of Calling source of Customers

Notation	parameteq	Description
$(M/M/1):(GD/\infty/\infty)$	P	poission arrival rate
	Q	exponential service rate
	R	Single server
	X	general disipline
	Y	Infinite number of custormers Permited in the system
	Z	Infinite size of Calling source

$(M/M/C):(GD/\infty/\infty)$	C	multi-server.
$(M/M/1):(GD/N/\infty)$	N	where 'N' represents finite no. of customers permited in the system
$(M/M/C):(GD/N/\infty)$	C	where 'C' represents multi server
$(M/M/1):(GD/N/N)$	N	finite size of calling source
$(M/M/C):(GD/N/N)$	C	multi server model

General Queueing model :-

where GD is first come first service

and λ - represents poisson arrival

μ - Exponential service rate

ρ - represents traffic intensity

$$\left[\rho = \lambda/\mu\right] < 1$$

The Formulae for Queuing Theory :—

(a) Average (or) Expected no. of units in the system (or) Mean Queue size (or) Length

$$\boxed{L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}}$$

(b) Average ~~to length~~

$$\boxed{L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu-\lambda)}}$$

3. expected waiting time in the system

$$\boxed{W_s = \frac{L_s}{\lambda}}$$

4. waiting time in the queue

$$\boxed{W_q = \frac{L_q}{\lambda}}$$

5. Expected waiting time of customer who has to wait

$$\boxed{(W/W > 0) = \frac{1}{\mu - \lambda}}$$

6. Expected length of non-empty Queue

$$\boxed{(L/L > 0) = \frac{\mu}{\mu - \lambda}}$$

7. probability of Queue size greater than N.

$$\boxed{(P > N) = \rho^N}$$

8. Studying state formula to find the probability of ~~formula~~ ~~exe~~ having N Customers in the system

$$\boxed{P_N = (1 - \rho) \rho^N}$$

9. probability of having atleast N Customers in the system at the time t.

$$\boxed{\int_0^t (\mu - \lambda) \, e^{-(\mu - \lambda)t} \, dt}$$

Problem :-

(1) In a railway station goods ride arrival 30 trains per day Assuming that the arrivals and service time follows exponesial Distribution a service time is 36 mits. find

(i) mean Queue size of the system (Queue length)

(ii) probability that the Queue size exceeds 10

(iii) If the Inputs of the trains Increases to the add average of 33 trains perday. what are the changes

The goods trains ride per day = 30

service train = 36 min

The arrival rate of goods trains λ = 30 Trains / days

$$= \frac{30}{24 \times 60} = \frac{30}{1440}$$

$$= \frac{1}{48}$$

Service Rate = $\frac{1}{36}$ min

Ls
Lq
Wq

Traffic Intensity (or) Utilization factor (or) System Business $\Big\}$ $\rho = \frac{\lambda}{\mu} = \frac{36}{48} = \frac{3}{4}$

(i) mean Queue size (or) Queue length

Sub numerical values in the formula

$$L_s = \frac{\lambda}{\mu - \lambda}$$

$$= \frac{\frac{1}{48}}{\frac{1}{36} - \frac{1}{48}} = \frac{\frac{1}{48}}{\left(\frac{48-36}{36 \times 48}\right)} = \frac{\frac{1}{48}}{\frac{1}{3 \times 48}}$$

$$= \frac{1}{48} \times \frac{3 \times 48}{1} \Rightarrow 3$$

(ii) probability that Queue size exceeds 10 $\quad 0.056$

$(P>N) = \rho^N = \rho^{10} = \left(\frac{3}{4}\right)^{10} = (0.75)^{10} = 0.056313514$

(iii) when the input increases 33 trains per day than the changes

The goods trains ride per day = 33 days $\quad \lambda = \frac{33}{24 \times 60} = 0.0229$

Service rate $(\mu) = \frac{1}{36}$

Traffic intensity (or) utilization factor (or) Business system $\rho = \frac{\lambda}{\mu} = \frac{0.0229}{\frac{1}{36}}$

$= 0.0229 \times 36$

$= 8.244$

mean Queue size (or) mean Queue length $(L_s) = \frac{\lambda}{\mu - \lambda} = \frac{0.0229}{\frac{1}{36} - 0.0229}$

$= \frac{0.0229}{0.0277 - 0.0229} = \frac{0.0229}{0.0048} = 4.770833$

problem :- 2

In a Supermart the average arrivals of customers is 10 in every 30 mins and follows poisson process the time per customer for purchase is 2.5 mins and follows exponential process.

what is the probability that the queue length exceeds 6

what is the expected time spend by a customer in the system [W_s]

The average arrivals of customers $\lambda = 10$

In every 30 mins

$\lambda = \frac{10}{30} = \frac{1}{3}$

Service time = 2.5 mins

Service rate $(\mu) = \frac{1}{2.5}$ mins

Traffice intensity (or) utilization factor (or) system Business $\Rightarrow \rho = \frac{\lambda}{\mu} = \frac{\frac{1}{3}}{\frac{1}{2.5}}$

$\Rightarrow \left[\frac{1}{3} \times 2.5\right] \Rightarrow \frac{2.5}{3}$

$\Rightarrow 0.8333$

(i) probability that the queue length

$$L_s = \frac{\lambda}{\mu - \lambda}$$

$$= \frac{\frac{1}{3}}{\frac{1}{2.5} - \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{3-2.5}{7.5}} = \frac{\frac{1}{3}}{\frac{0.5}{7.5}} \Rightarrow \frac{1}{3} \times \frac{7.5}{0.5}$$

$$\Rightarrow \frac{7.5}{1.5} = 5$$

$$L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$= \frac{\left(\frac{1}{3}\right)^2}{\left(\frac{1}{2.5}\right)\left(\frac{1}{2.5} - \frac{1}{3}\right)} = \frac{\frac{1}{6}}{\left(\frac{1}{2.5}\right)\cdot\left(\frac{0.5}{7.5}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{0.5}{18.7500}} = \frac{1}{6} \times \frac{18.75}{0.5}$$

$$= \frac{18.75}{3} = 6.25$$

(ii) The expected time spend by a customer in the system

$$W_s = \frac{L_s}{\lambda} = \frac{5}{1/3}$$

$$= 5 \times 3$$

$$= 15$$

3. A public telephone booth arrival an average 15 per hour a phone Call on average takes 3 mins only one phone is available
1. Expected number of Columns in the book [non empty]
2. the ideal time of telephone booth

In the telephone booth arrival time $\lambda = 15$ per hour $\Rightarrow \frac{15}{60}$

$\Rightarrow \frac{1}{4}$ per min

the duration of telephone call $= 3$ mins

The telephone Call Rate $\mu = \frac{1}{3}$ per min

Traffic intensity, utilization factor, System business

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$

Expected number of Columns in the books [non empty]

$$(L/L>0) = \frac{\mu}{\mu-\lambda} = \frac{\frac{1}{3}}{\frac{1}{3}-\frac{1}{4}} = \frac{\frac{1}{3}}{\frac{4-3}{12}} = \frac{\frac{1}{3}}{\frac{1}{12}}$$

$$= \frac{12}{3} = 4$$

Ideal time of telephone booth $= 1 -$ System business

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

4. A TV repair man finds time spent on his job with mean 80 minutes and follows exponential distribution. The goods arrived at a mean of 10 per 8 hour day and follows poisson distribution 1. what is the expected Ideal time
2. How many jobs are ahead of just brough in

A TV repairman time-spent on his job

The service rate $\mu = \frac{1}{30}$ per mints

$= \frac{6\cancel{0}}{3\cancel{0}} = 2$ per min

The arrival rate of [TV's] goods $= \lambda = \frac{10}{8}$ per day

System business, Traffic intensity, utilization factor

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{10}{8}}{2} \Rightarrow \frac{\cancel{10}^{5}}{8} \times \cancel{2} \Rightarrow \frac{5}{8}$$

(i) Expected Ideal of time of repair shop

$$\Rightarrow 1 - \rho$$

$$\Rightarrow 1 - \frac{5}{8} \Rightarrow \frac{8-5}{8}$$

$$\Rightarrow \frac{3}{8}$$

(ii) Jobs are ahead of just brough in

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{\frac{10}{8}}{2 - \frac{10}{8}} = \frac{\frac{10}{8}}{\frac{16-10}{8}} = \frac{\frac{10}{\cancel{8}}}{\frac{6}{\cancel{8}}} = \frac{10}{6} = \frac{5}{3}$$

(or)

$$L_s = \frac{\rho}{1-\rho} = \frac{\frac{5}{8}}{1 - \frac{5}{8}} = \frac{\frac{5}{\cancel{8}}}{\frac{8-5}{\cancel{8}}}$$

$$\boxed{L_s = \frac{5}{3}}$$

5. Cars arrives at pertorol pump station at an average 10 per hour the service time is 3 minutes. find the average no of cars in the system

(ii) Average waiting time in the Queue

(iii) Average Queue length.

(iv) probability that the number of cars in the system is '2'

A Cars arrives at pertol pump station average 10 per hour

$\cancel{\lambda = \frac{1}{10} = \frac{60}{60} = 6 \text{ min}}$ $\boxed{\lambda = 10}$

The service time $\mu = 3$, The service rate $\mu = \frac{1}{3}$ min

$= \frac{1}{3} \times 60$

$= 20$

Traffic intensity, utilization factor, System business $\Big\}$ $\rho = \frac{\lambda}{\mu}$

$= \frac{10}{20} = \frac{1}{2}$

(i) Average no. of cast in the system

Ls=1

$$Ls = \frac{\rho}{1-\rho} = \frac{\frac{1}{2}}{1-\frac{1}{2}} \Rightarrow \frac{\frac{1}{2}}{\frac{2-1}{2}} \Rightarrow \frac{\frac{1}{\cancel{2}}}{\frac{1}{\cancel{2}}}$$

$\boxed{Ls = 1}$

(ii) Average waiting time in the Queue.

$$Wq = \frac{Lq}{\lambda} \Rightarrow \frac{\frac{1}{2}}{10} \Rightarrow 0.05 \text{ mins}$$

$\Rightarrow 0.05 \times 60$

$\Rightarrow 3 \text{ min}$

(iv) probability of the no. of cars in the system is '2'

$$P_N = (1-\rho) \cdot \rho^N$$

$$= \left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 \Rightarrow \left(\frac{1}{2}\right)\cdot\left(\frac{1}{4}\right)$$

$$P_N = \frac{1}{8}$$

(iii) Average Queue length

$$L_q = L_s - \frac{\lambda}{\mu} = 1 - \frac{1}{2}$$

$$= \frac{2-1}{2} = \frac{1}{2}$$

6. In a phone booth inter arrival time is 12 mins the length of a phone call is 4 mins

i) what is the probability that a fresh arrival will not have to wait for the phone

(ii) what is the probability an arrival will have to wait more than 10 mins before the phone

(iii) what is the average length of the Queue for time to time

The phone booth inter arrival time $\lambda = 12$ mins

arrival rate $\lambda = \frac{1}{12}$ min

The service time is 4 mins

The service rate $\mu = \frac{1}{4}$ ~~per~~ min

Traffic intensity (or) Utilization factor (or) System business $\Bigg\}$ $\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{4}{12} = \frac{1}{3}$ min

(i) probability that the fresh arrival will not have to wait for the phone

$$\Rightarrow 1 - \rho$$

$$\Rightarrow 1 - \frac{1}{3} \Rightarrow \frac{3-1}{3}$$

$$\Rightarrow \frac{2}{3}$$

(ii) probability that the arrival will have to wait for the phone atleast 10 mins

0.809

$$\Rightarrow \int_0^t (\mu-\lambda)\, e^{-(\mu-\lambda)t}\, dt$$

$$\Rightarrow \int_0^{10} \left(\frac{1}{4}-\frac{1}{12}\right) e^{-\left(\frac{1}{4}-\frac{1}{12}\right)^{10}} dt \quad \left(\frac{1}{2}\right) e^{-\left(\frac{1}{6}\right)^{10}}$$

(iii) ~~the~~ average length of Queue for time to time

Expected length of the Queue non empty $= \frac{\mu}{\mu-\lambda} = \frac{\frac{1}{4}}{\frac{1}{4}-\frac{1}{12}}$

H.W

7. Arrivals in a telephone booth is 10mins duration of Call 3mins i) what is the probability that a person arriving will have to wait

(ii) Average no. of units each persons in the system

(iii) estimate a ~~fa~~fraction of a day a phone will be use

(iv) what is the probability that it will take more than 10mins all together to wait for the phone and Complete the call

Model :- 2 Multi-channel syst

In this model in the place of single service channel there are "C" parallel channels. C servers are providing service to C customers. Customers will arrive at a rate of λ on average and follows possion distribution the service rate is μ on average and follows exponensial distribution.

If there are more than 'C' customers in the system, all servers in will be busy and means service rate is equal to "$C * \mu$".

If there are 'n' customers, where $n<c$ than the service rate is "$n * \mu$". and (c-n services will be Ideal thus we have

(i) $\mu = \mu_1$ $\mu_2 = 2 \cdot \mu_1$ $\mu_3 = 3 \cdot \mu \cdots \cdots \mu_n = n * \mu$ for $(n<c)$

(ii) If $n>c$, then c customers on serve and $n-c$ (customers will be in the queue and $\mu_n = C \cdot \mu$ for $(n \geq c)$

(iii) arrival rate is same as $\lambda_n = \lambda$ for all values of 'n'.

Formulae applied :-

The study state probabilites as follows

$$P_1 = \frac{\lambda}{\mu} \cdot P_0 \qquad P_2 = \frac{\lambda^2}{\mu_2 * \mu} P_0$$

$$P_3 = \frac{\lambda^3}{\mu_3 \cdot \mu_2 \cdot \mu} \cdot P_0$$

$$\Rightarrow P_n = \frac{\rho^n}{n!} \cdot P_0 \text{ for } (n<c)$$

$$\Rightarrow P_n = \frac{\rho^n}{C^{n-c} \cdot C!} P_0 \text{ for } (n \geq c)$$

probability that zero Customers :-

1. $P_0 = \dfrac{1}{\sum_{n=0}^{c-1} \dfrac{\rho^n}{n!} + \dfrac{\rho^c}{c!} * \dfrac{c \cdot \mu}{(c \cdot \mu - \lambda)}}$

Note :- This Result is valid only. when

$\frac{\rho}{c} < 1$ (or) $\frac{\lambda}{c \cdot \mu} < 1 \Rightarrow \lambda < c \cdot \mu$

2. probability that the arrival has to wait

$$P(n \geq c) = \frac{\rho^c}{c!} \left[\frac{c \cdot \mu}{c\mu - \lambda} \right] * P_0$$

3. probability that an arrival will not the wait

$$P(n \geq c) = 1 - P(n < c)$$

4. Average Queue length (L_q)

$$L_q = \frac{\rho^{c+1} \cdot \mu^2}{(c-1)!\,(c\mu - \lambda)^2} * P_0$$

5. Average no. of customers in the System

$$L_s = L_q + \rho$$

6. waiting time of a Customer in the queue

$$W_q = \frac{L_q}{\lambda}$$

7. average waiting time of a Customer in the System

$$W_s = W_q + \frac{1}{\mu}$$

8. average no. of idle servers or no. of servers remaing idle

$$c - \rho \Rightarrow c - \frac{\lambda}{\mu}$$

(1) A super market has two sails girls the service time in 4 min on an average a arrival wait is 10 per hour

(a) The probability that an arrival has to wait

(b) Expected pertentage of idle time for each girl

(c) Expected waiting of a Customer in the system

By data arrival wait $\lambda = 10$ per hour $\Rightarrow \frac{1}{6}$ per min

Service time $= 4$ mins

Serivce rate $\mu = \frac{1}{4}$ per min

Traffic intensity, system business utilization factor $\Big\}$ $\rho = \frac{\lambda}{\mu} = \frac{2}{3}$

No. of servers $C = 2$

probability that zero customers in the system

$$P_0 = \frac{1}{\sum_{n=0}^{C-1} \frac{\rho^n}{n!} + \frac{\rho^C}{C!} * \frac{C \cdot \mu}{(C\mu - \lambda)}}$$

$$P_0 = \frac{1}{\sum_{n=0}^{1} \frac{\rho^n}{n!} + \frac{\rho^2}{2!} * \frac{2 * \frac{1}{4}}{\left(2 \times \frac{1}{4} - \frac{1}{6}\right)}}$$

$$P_0 = \frac{1}{1 + \frac{2}{3} + \frac{\left(\frac{2}{3}\right)^2}{2} * \frac{3}{2}}$$

$\Rightarrow 1 + \frac{2}{3} + \frac{4}{18} * \frac{3}{2}$

$\Rightarrow 1 + \frac{2}{3} + \frac{2}{3\cancel{9}}\left[\frac{\cancel{3}}{\cancel{2}}\right]$

$\Rightarrow 1 + \frac{2}{3} + \frac{1}{3}$

$\Rightarrow \frac{3+2+1}{3}$

$\Rightarrow \frac{\cancel{6}^{2}}{\cancel{3}_{1}} \quad \left[\frac{1}{\frac{2}{1}}\right] \times \frac{1}{2}$

$\Rightarrow \frac{1}{2}$

$P_1 = \frac{\lambda}{\mu} * P_0 = \frac{1}{3}$

$$P_1 = \frac{\frac{1}{6}}{\frac{1}{4}}$$

$= \frac{1}{6} \times \frac{4}{1} \Rightarrow \frac{\cancel{4}^{2}}{\cancel{6}} * \frac{1}{\cancel{2}} \Rightarrow \frac{\cancel{2}}{\cancel{6}} \Rightarrow \frac{1}{3}$

(a)

probability that an arrival has to wait

$$P(n \geq 2) = 1 - P_0 - P_1 = \frac{1}{6} \left[1 - \frac{1}{2} - \frac{1}{3}\right] = \frac{6-3-2}{6} = \frac{1}{6}$$

(b) The expected no. of idle girls or Counters $= c - \rho \Rightarrow 2 - \frac{2}{3} = \frac{4}{3}$ idle

Probability that a girl is idle $= \dfrac{\text{expected no. of idle girls}}{\text{Total no. of girls}} = \dfrac{\frac{4}{3}}{2} = \frac{4}{6} = \frac{2}{3} = 0.67 = 67\%$

Percentage of idle time for each girl $= 67\%$.

Average Queue length

$$L_q = \frac{\left(\frac{2}{3}\right)^{2+1}\left(\frac{1}{4}\right)^2}{(2-1)^1\left(2 \cdot \frac{1}{4} - \frac{1}{6}\right)^2} * \frac{1}{2}$$

$$= \frac{\left(\frac{2}{3}\right)^3\left(\frac{1}{4}\right)^2}{1\left(\frac{1}{2} - \frac{1}{6}\right)^2} * \frac{1}{2} \Rightarrow \frac{\left(\frac{8}{27}\right)\left(\frac{1}{16}\right)}{\left(\frac{1}{3}\right)^2} * \frac{1}{2}$$

$$\Rightarrow \frac{\frac{1}{\cancel{54}\ 6}}{\frac{1}{9}} * \frac{1}{2} \Rightarrow \frac{1}{12}$$

(c) Expected waiting of a Customer in the system

$$W_s = W_q + \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda} \Rightarrow \frac{1/12}{1/6} \Rightarrow \frac{1}{2} \times \frac{6}{1} = \frac{1}{2}$$

$$W_s = W_q + \frac{1}{\mu}$$

$$\Rightarrow \frac{1}{2} + \frac{1}{\frac{1}{4}}$$

$$\Rightarrow \frac{1}{2} + 1 \cdot \frac{4}{1} \Rightarrow \frac{1+8}{2} \Rightarrow 9/2$$

Theae are 4 Counters in at a check point for a verfication of passport the arrival rate of persons is λ and service date on average is $\frac{\lambda}{2}$ find the expected queue length at the check point. (L_q) P_0

By Data arrival wait $= \lambda$

Service rate $\mu = \frac{\lambda}{2}$

system business $\rho = \frac{\lambda}{\mu} = \frac{\lambda}{\lambda/2} = \lambda \times \frac{2}{\lambda} = 2$

no. of servces $C = 4$

$$\Rightarrow P_0 = \frac{1}{\sum_{n=0}^{4-1} \frac{\rho^n}{n!} + \frac{\rho^4}{4!}\left(\frac{4 \cdot \mu}{4\mu - \lambda}\right)}$$

$$= \sum_{n=0}^{3} \frac{\rho^n}{n!}$$

$$= \frac{\rho^3}{3 \times 2 \times 1} + \frac{\rho^2}{2 \times 1} + \frac{\rho^1}{1}$$

$$= \frac{\rho^3}{6} + \frac{\rho^2}{2} + \frac{\rho}{1} + \frac{\rho^0}{0}$$

$$= 1 + \rho + \frac{\rho^2}{2} + \frac{\rho^3}{6} + \frac{\rho^4}{24}\left(\frac{4 \cdot \mu}{4 \cdot \mu - \lambda}\right)$$

$$= 1 + 2 + \frac{(2)^2}{2} + \frac{(2)^3}{6} + \frac{(2)^4}{24}\left[\frac{4\left(\frac{\lambda}{2}\right)}{4\left(\frac{\lambda}{2}\right) - \lambda}\right]$$

$$= 1 + 2 + \frac{4}{2} + \frac{8}{6} + \frac{16}{24}\left[\frac{2\lambda}{2\lambda - \lambda}\right]$$

$$= 1 + 2 + 2 + \frac{4}{3} + \frac{2}{3}\left(\frac{2\cancel{\lambda}}{\cancel{\lambda}}\right)$$

$$= 1 + 2 + 2 + \frac{4}{3} + \frac{4}{3}$$

$$= \frac{3 + 6 + 6 + 4 + 4}{3}$$

$$P_0 = \frac{23}{3} \Rightarrow \frac{1}{\frac{23}{3}} \quad P_0 = \frac{3}{23}$$

$$L_q = \frac{e^{c+1} \cdot \mu^2}{(c-1)!\,(c-\mu-\lambda)^2} P_0 = \frac{2^{4+1}\left(\frac{\lambda}{2}\right)^2}{(4-1)!\,\left(4\left(\frac{\lambda}{2}\right)-\lambda\right)^2} \cdot \frac{3}{23}$$

$$\Rightarrow \frac{2^5\left(\frac{\lambda}{2}\right)^2}{3!\,(2\lambda-\lambda)^2} \cdot \frac{3}{23}$$

$$= \frac{2\times2\times2\times2\times2\left(\frac{\lambda^2}{4}\right)}{3!\,(2\lambda-\lambda)^2} \cdot \frac{3}{23} \Rightarrow \frac{2\times2\times\cancel{2}\times\cancel{2}\times\cancel{2}\left(\frac{\cancel{\lambda^2}}{\cancel{4}}\right)}{\cancel{3}\times\cancel{2}\,(\cancel{\lambda^2})} \cdot \frac{\cancel{3}}{23}$$

$$= \frac{4}{23}$$